Classical Mechanics I

Homework Set 1

1. In class, we found the velocity of a block sliding down an incline under a kinetic friction force and an air resistance force equal to kmv. Solve for the velocity of the block again, except use kmv^2 as the expression for air resistance. Using the same numerical values as the in class example, what is the maximum speed of the block? How does your answer compare the v_{max} we found in class when we used kmv?

   Note: There are several forms for the solution to the integral involving v^2, including natural logs and hyperbolic tangents. For consistency for everyone, use the hyperbolic tangent solution expression.

   $$\int_0^v \frac{dv}{a - bv^2} = \frac{1}{\sqrt{ab}} \tanh^{-1}\left(v\sqrt{\frac{b}{a}} \right)$$

2. **Consider a particle of mass m fired vertically at a speed of v_o in a constant gravitational field, g. For the same initial velocities, compare the times required for the projectile to reach its maximum height:
   a. For no air resistance
   b. For air resistance of the form kmv

   Show that the time involving air resistance will always be less than the time for no air resistance by taking a series expansion of the *ln* in your answer for part *b* and keeping the first two terms.

   $$\ln(1 + x) \approx x - \frac{1}{2}x^2$$

3. An automobile driver traveling down an 8% grade slams on his brakes and skids 30 *m* before hitting a parked car. A lawyer hires an expert who measures the coefficient of kinetic friction between the tires and the road to be $\mu_k = 0.45$.
   a. How far would the car skid if traveling at 25 *mph*?
   b. What is the estimated minimum speed of the car (*in mph*) based on the evidence provided?
   c. Is the lawyer correct to accuse the driver of exceeding the posted 25 MPH speed limit? Explain

4. **A particle of mass m is released from rest (starting from $y = 0$) and falls under the influence of gravity and air resistance. Find and expression for $y(v)$ when the air resistance is equal to:
   a. $f(v) = \alpha v$
   b. $f(v) = \beta v^2$

   Hint: When integrating, use the chain rule: $\ddot{y} = \frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = v\frac{dv}{dy}$

5. **A boat with an initial speed v_o is launched on a lake with no motor. The boat is slowed by the water by a force $F(v) = -\alpha e^{\beta v}$.
   a. Find an expression for the speed of the boat, $v(t)$
   b. Find an expression for the time it takes the boat to stop

6. When a baseball flies through the air, the ratio f_{quad}/f_{lin} of the quadratic to the linear drag force is given by

$$\frac{f_{quad}}{f_{lin}} = \left(1.6\,x10^{3}\,\tfrac{s}{m^2}\right)Dv$$

Given that the baseball has a diameter of 7 *cm*,

   a. Find the approximate speed v at which the linear and quadratic drag forces are equally important?
   b. For what speeds is it a good approximation to ignore the linear term and to treat the drag force as purely quadratic?
   c. Repeat the previous two questions for a beach ball with a diameter of 70 *cm*.

7. **A mass m has a velocity v_o at time $t = 0$ and coasts along the x-axis in a medium where the drag force has the form $f(v) = kmv^{3/2}$. Find $v(t)$ and the time (if any) when it will come to rest.

8. Assume each is spherical and dropped from a great height. Find the terminal speeds of:
   a. A steel ball bearing of diameter 3 *mm*
   b. A 16 *lb* steel shot
   c. A 200 *lb* parachutist in free fall in the fetal position

   In all three cases, you can assume the drag force is purely quadratic ($\gamma D^2 v^2$) and $\gamma = 0.25\ Ns^2/m^4$. The density of steel is about 8000 kg/m^3 and you can treat the parachutist as a sphere of density 1000 kg/m^3.

   Hint: In free fall, the vertical acceleration ($\ddot{y}$) is equal to zero making solving for v in the equation of motion very easy. Also remember to make good use of the mass density expression, $\rho = m/V$

9. **A puck of mass m is kicked up an incline with a slope θ and initial speed v_o. Assume there is friction between the puck and the incline with a coefficient of kinetic friction of μ_k and air resistance with a magnitude $f(v) = kmv^2$.
   a. Find $v(t)$ as it moves up the incline
   b. Determine t_{stop}, the time it takes to come to rest

   Hint: If you use the same numerical values as in problem 1 along with v_o = 10 m/s in part (b), you should get 1.60 *sec* time to stop. Be sure your calculator is in radians when evaluating arc-trig functions. The following integral will also prove useful.

$$\int \frac{dv}{a + bv^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(v\sqrt{\frac{b}{a}}\right) + C$$