Classical Mechanics I

Homework Set 2

1. A particle of mass $m = 1\ kg$ is subjected to a one-dimensional force $F(t) = kte^{-\alpha t}$, where $k = 1$ N/s and $\alpha = 0.5\ s^{-1}$. If the particle is initially at rest:
   a. Calculate expressions for $x(t)$, $v(t)$ and $a(t)$
   b. Plot your results from part (a) for $t = 0$ to 20 sec.

2. A particle of mass m has speed $v = \alpha/x$, where x is its displacement. Find the force $F(x)$ responsible.

3. The speed of a particle of mass m varies with the distance x as $v(x) = \alpha x^{-n}$. Assume $v(0) = 0$.
   a. Find the force $F(x)$ responsible
   b. Determine $x(t)$
   c. Determine $F(t)$

4. **A particle is under the influence of a force $F(x) = -kx + kx^3/\alpha^2$, where k and a are constants and k is positive.
   a. Determine $U(x)$
   b. Plot $U(x)$ (*with* $k = 100\ N/m$ *and* $\alpha = 0.5\ m$) and discuss the motion for different values of E
   c. Determine the equilibrium point locations for $U(x)$ in terms of α. Are they stable or unstable?
   d. What would you observe about the motion of the particle if $E = U(\pm\alpha) = (1/4)k\alpha^2$?

5. Consider a particle moving in the region $x > 0$ under the influence of the potential

$$U(x) = Uo\left(\frac{a}{x} + \frac{x}{a}\right)$$

   where $U_o = 1$ J and $a = 2\ m$.

   a. Plot the Potential $U(x)$
   b. Find the equilibrium points of $U(x)$ and indicate where they are maxima or minima.

6. If a particle's potential energy is $U(\mathbf{r}) = k\,(x^2+y^2+z^2)$, where k is a constant, what is the force $F(\mathbf{r})$ on the particle? Express your answer in terms of $\mathbf{r}$.

7. A shell of mass m traveling horizontally with speed v_o due North explodes into two equal mass fragments. It is observed that after the explosion, one fragment is traveling vertically up with speed v_o. What is the velocity of the other fragment? Define a coordinate system such that North is in the $\hat{\boldsymbol{y}}$ direction, East in is in the $\hat{\boldsymbol{x}}$ direction, and $\hat{\boldsymbol{z}}$ is upward. Assume the explosion occurs when the shell is at the origin.

8. According to special relativity, a particle of rest mass m_o accelerated in one dimension by a force F obeys the equation of motion $F = dp/dt$. Here, $p = m_o v/(1 - v^2/c^2)^{1/2}$, which is the particles relativistic momentum, which reduces to $m_o v$ when $v^2/c^2 << 1$. For the case of constant F and initial conditions $x(0) = 0$ and $v(0) = 0$,
    a. Find $x(t)$ and $v(t)$
    b. Plot your expression for $v(t)$ from $t = 0$ to 4×10^8 sec using $F/m_o = 10\ m/s^2$.
    c. How much time (*in yrs*) is required for the particle to reach 50% the speed of light? How much time (*in yrs*) is required to reach 99% the speed of light?

9. Find the partial derivatives with respect to x, y and z for the following functions in which a, b and c are constants:
    a. $f(x,y,z) = ax^2 + bxy + cy^2$
    b. $f(x,y,z) = \sin(axyz^2)$
    c. $f(x,y,z) = ae^{xy/z^2}$

    To test your answers, use $a = 1$, $b = 2$ and $c = -3$ for the constants and evaluate the partial derivative answers at (7,1,-3).