Classical Mechanics I

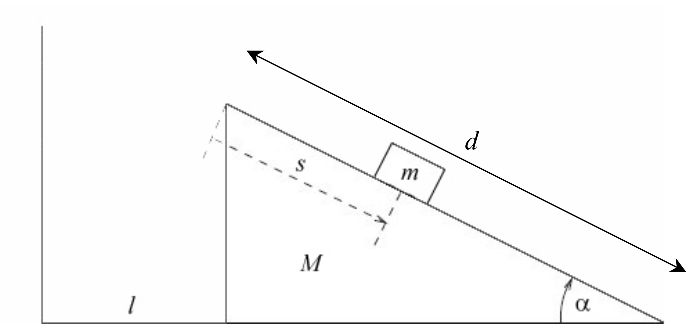
Homework Set 6

Do NOT solve any of the equations of motion unless specifically asked to do so!

Each problem MUST have a properly drawn and labeled picture!

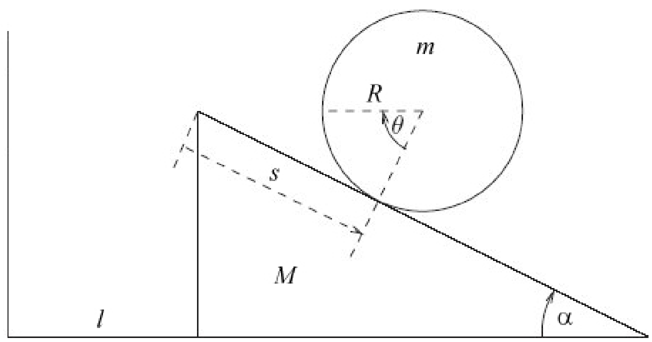
1. A block of mass m slides down a movable inclined plane of mass M, which makes an angle α with the horizontal. Find the coupled equations of motion for both masses in this system (*one for the block and one for the wedge*).

   Hint: Let d be the length of the incline. Use l and s as the generalized coordinates. Since both are solid objects, pick a single point on M and m to use for converting each from cartesian to generalize coordinates.



2. A hoop of mass m and radius R rolls without slipping down a movable inclined plane of mass M, which makes an angle α with the horizontal. Find the equations of motion for both masses in this system (*one for the hoop and one for the wedge*).

   Hint: Let d be the length of the incline. Use l and s as the generalized coordinates.

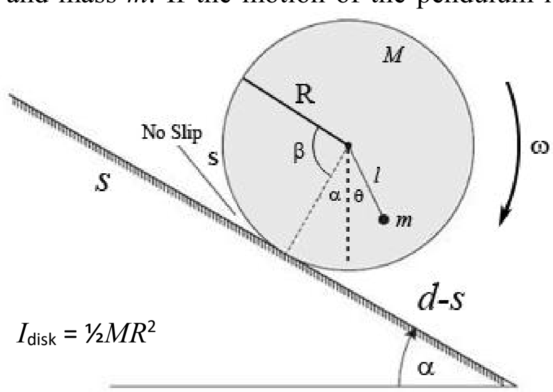


$I_{\text{hoop}} = mR^2$

3. A disk of mass M and radius R rolls without slipping down a fixed inclined plane, which makes an angle α with the horizontal. The disk has a short, weightless axle of negligible radius from which is suspended a simple pendulum of length $l < R$ and mass m. If the motion of the pendulum is parallel to the disk, find the coupled equations of motion for this system (*one for the disk and one for the pendulum*).

   Hint: Let d be the length of the incline. Use s and θ as the generalized coordinates. The following trig identity will be helpful in reducing the Lagrangian:

   $$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

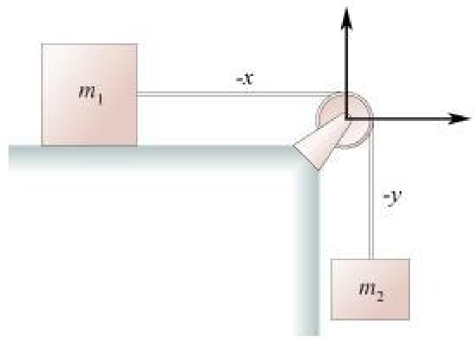


$I_{\text{disk}} = ½MR^2$

4. Consider a single mass m moving in two dimensional plane with a potential energy $U(x,y) = \frac{1}{2}kr^2$, where $r^2 = x^2 + y^2$.
   a. Determine the equations of motion for this system using Cartesian coordinates.
   b. What can you determine about the motion of the mass m from your equations?

(**Note**: This is the potential energy of an ion in an "ion trap", which can be used to study the properties of individual atomic ions.)

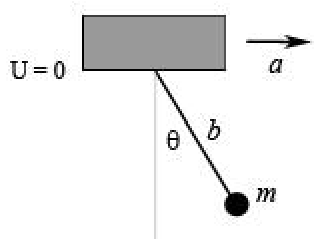
5. Two blocks are connected by a uniform string of length l and negligible mass. One block is placed on a smooth, frictionless horizontal surface and the other hangs over the side, with the string passing over a frictionless pulley at the corner of the surface. If $m_1 = m_2 = m$ and using x as your system variable,
   a. What is the acceleration of the masses ($\ddot{x}$)?
   b. How does your answer in part (a) compare to the result found in University Physics I using Forces?



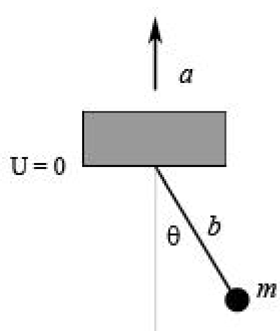
$$a = \left(\frac{m_1}{m_1 + m_2}\right) g$$

(**Note**: Because of the choice of origin, $l = -x - y$)

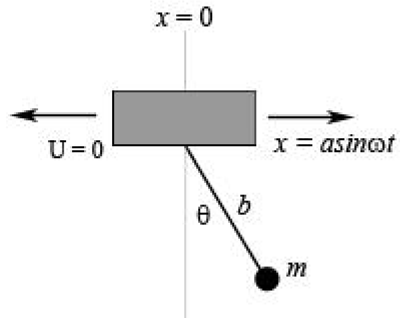
6. A simple pendulum of length b and mass m is attached to a massless support block that starts moving horizontally to the right from rest with constant acceleration a. Determine the equation of motion for the pendulum.



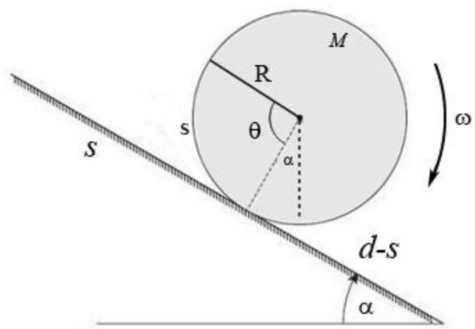
7. A simple pendulum of length b and mass m is attached to a massless support block starts moving vertically upward form rest with constant acceleration a. Determine the equation of motion for the pendulum.



8. A simple pendulum of length b and mass m is attached to a massless support block oscillating horizontally such that $x = a\sin\omega t$. Determine the equation of motion for the pendulum.



9. Find the linear acceleration ($\ddot{s}$) of a solid uniform sphere rolling without slipping down an inclined plane, which makes an angle α with the horizontal.



$I_{\text{sphere}} = \frac{2}{5}MR^2$

10. A smooth wire is bent into the shape of a helix, with cylindrical polar coordinates $\rho = R$ and $z = k\varphi$, where R and k are constants and the z axis is vertically upward (*and gravity down*). Using z as your generalized coordinate:
    a. Determine the equation of motion for a bead of mass m sliding on the wire after being released from a height h.
    b. In the limit that R → 0, what is $\ddot{z}$? Does this make sense?