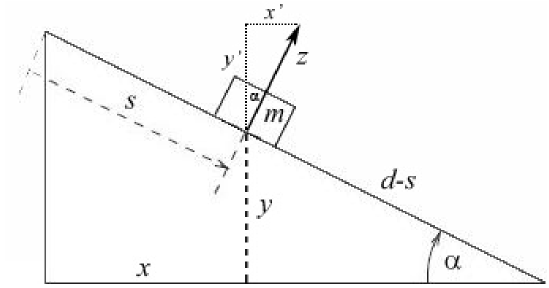
**Classical Mechanics I**

*Homework Set 7*

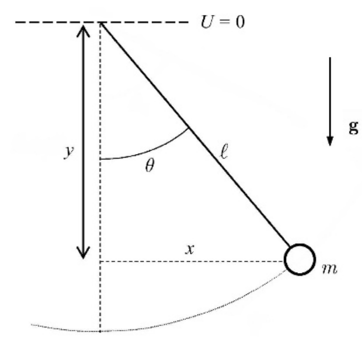
***Do NOT solve any of the equations of motion unless specifically asked to do so!***

1. A block of mass $m$ slides down an inclined plane, which makes an angle $\alpha$ with the horizontal. Determine the expression for the Lagrangian and find:
   *a.* The acceleration of the system ($\ddot{s}$)
   *b.* The forces of constraint acting on mass $m$ as it slides down the incline (*can you guess what the forces might be*?).

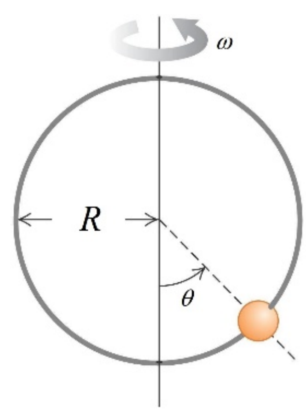



**Note:** In this problem, we have to use an additional variable ($z$) to model the behavior of the block on or off the incline. The equation of constraint is $z = 0$ so that the block remains on the surface, but do NOT assume or apply this condition when setting up your Lagrangian. Only sub in after you have found your Euler-Lagrange with Constraints (ELC) equations.

2. Consider a simple pendulum of length $L$ and mass $m$. Using $l$ and $\theta$ as generalized coordinates, find:
   *a.* The equation of motion for the system
   *b.* The forces of constraint on mass $m$ as it swings




3. A bead of mass $m$ is free to slide along a frictionless hoop of radius $R$ and mass $M$. The hoop rotates with constant angular speed $\omega$ around a vertical axis as shown in the figure at right. Find:
   *a.* The equation of motion for the bead in terms of $\theta$.
   *b.* For a given $\omega$, what are the equilibrium positions ($\theta_o$)?




**Hint:** Use Spherical Coordinates with $\omega = \dot{\varphi}$ = constant. Also, the following relations will prove helpful when determining velocity terms:

$$\sin(\pi - \theta) = \sin\theta$$

$$\cos(\pi - \theta) = -\cos\theta$$

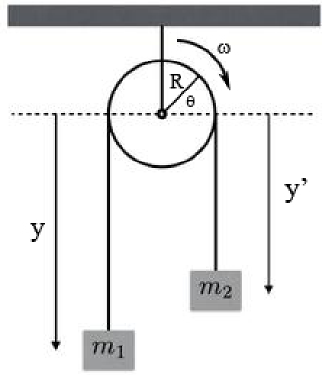
**Note:** There is NO explicit rotational kinetic energy associated with the bead itself because (1) it is only sliding up the hoop, not rotating on the wire and (2) its rotational energy is accounted for in the spherical coordinate transformations. The moment of inertia for the hoop is $MR^2$. Equilibrium occurs when $\ddot{\theta}$ and $\dot{\theta} = 0$.

4. Consider a simple Atwood's Machine, where $m_2 > m_1$ and the length of the string is $l = y + y' + \pi R$. Determine

   *a.* The Lagrangian for the system.

   *b.* The equation of motion for the system in terms of $y$

**Hint:** Be sure to include the rotational KE of the pulley. The moment of inertia of the pulley is $\frac{1}{2} MR^2$.

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