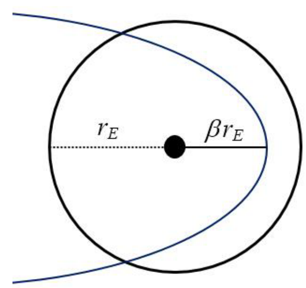
Classical Mechanics I

Homework Set 8

1. One particle moves around a second, more massive particle, under the influence of their mutual gravitational force in a circular orbit with a period of τ.
   a. If the moving particle suddenly stopped in its orbit, show that the two particles would gravitate together and collide after a time $\tau / 4\sqrt{2}$.
   Hint: Use Conservation of Energy
   b. How long would it take the Earth (*in days*) to fall into the Sun if it were suddenly stopped in its orbit?
   c. Find the orbital period of the moon about the Earth (*assume circular*).
   d. How long would it take the Moon (*in days*) to fall into the Earth if it were suddenly stopped in its orbit?

2. Consider a comet moving in a parabolic orbit in the plane of Earth's orbit. If the distance of closest approach of the comet to the Sun is βr_E, where r_E is the radius of Earth's (*assumed circular*) orbit and $\beta < 1$,
   a. If τ_E is the period of the Earth's orbit, show that the time the comet spends within the orbit of Earth is given by

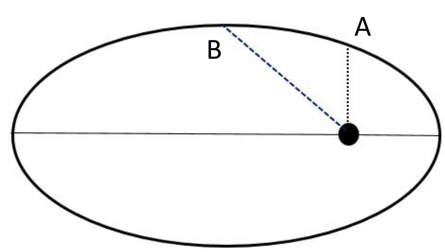
$$\frac{\tau_E}{3\pi}(1+2\beta)\sqrt{2(1-\beta)}$$



   b. If the comet approaches the Sun to the distance of the perihelion of Mercury ($\beta = 0.387$), how many days is it within Earth's orbit?

3. ** The orbit of a particle moving in a central field is a circle passing through the origin according to $r = r_o \cos(\theta)$. Show that the force law required for this type of orbit is an inverse-fifth power.

4. An Earth satellite has a perigee of 300 *km* and an apogee of 3500 *km* above Earth's surface. How far is the satellite above the Earth when
   a. it has rotated 90° around the Earth from perigee (*point* A)
   b. it has moved halfway from perigee to apogee (*point* B)



5. An Earth satellite has a speed of 28070 *km*/*hr* when it is at its perigee of 220 *km* above Earth's surface. Find the satellite's
   a. apogee distance (*in km*) above the Earth's surface
   b. speed at apogee (*in m*/*s*)
   c. period of revolution (*in hours*)

6. What is the minimum escape velocity of a spacecraft from the surface of the moon?

7. Calculate the minimum Δv required to place a satellite already in Earth's heliocentric orbit (*assumed circular*) into the orbit of Venus (*also assumed circular and coplanar with Earth*) using a Hohmann transfer. Consider only the gravitational attraction of the Sun. What is the time of flight for such a trip (*in days*)?

8. A spacecraft is parked in a circular orbit 200 *km* above the Earth's surface. We want to use a Hohmann transfer to send the spacecraft to the Moon's orbit.
   a. What is the total Δv required?
   b. How long would the transfer take (*in days*)?

9. Alpha Centauri is the nearest star system, about 4 light years from Earth. An ion rocket has been built to travel to Alpha Centauri and currently sits in a circular orbit about earth with an orbital speed of 32.9 *km*/*hr*. The rocket will burn all its fuel in about 100 hours, producing an ion exhaust velocity of one-tenth the speed of light. Assume the initial mass of the fuel be twice that of the payload (*ignore the mass of the rocket itself*). How (*in years*) will it take the rocket to reach Alpha Centauri? (*The speeds are small enough that you can ignore the effects of special relativity*.)

10. Assume a satellite is being launched from Cape Canaveral to a circular LEO with an orbital speed of 8 *km*/*s*. Satellites are launched toward the east to take advantage of Earth's rotation, which reduces the actual speed necessary to reach orbit. For a point on the Earth near the equator, the rotational speed of the earth is approximately $R_E\omega_E$, which is about 0.5 *km*/*s*. Assuming the rocket fuel ejection speed is 3 *km*/*s*:
    a. Ignoring gravity, find the payload to fuel ratio (m_o/m) for this type of rocket to reach the necessary orbital speed after launch.
    b. How does this ratio compare to the value in the example worked in class of 14.4?
    c. If an original estimate for the necessary rocket fuel would cost $1.4 million at a mass ratio of 14.4, how much is saved by using the Earth's rotation to help reach orbit?

Planetary Data:
Radius of the Earth: 6371 *km*
Mass of the Earth: 5.97×10^{24} *kg*
Radius of the Moon: 1737 *km*
Mass of the Moon: 7.35×10^{22} *kg*
Radial Distance to the Moon: 3.825×10^{8} *m*
Mass of the Sun: 1.99×10^{30} *kg*
Orbital Radius of Mercury: 0.387 A.U. (5.791×10^{10} *m*)
Orbital Radius of Earth: 1 A.U. (1.496×10^{11} *m*)
Orbital Radius of Venus: 0.723 A.U. (1.082×10^{11} *m*)