2-16. The only force which is applied to the article is the component of the gravitational force along the slope: *mg* sin *α*. So the acceleration is *g* sin *α*. Therefore the velocity and displacement along the slope for upward motion are described by:

$$v = v_0 - (g \sin\alpha)t \tag{1}$$

$$x = v_0 t - \frac{1}{2}(g \sin\alpha)t^2 \tag{2}$$

where the initial conditions $v(t=0) = v_0$ and $x(t=0) = 0$ have been used.

At the highest position the velocity becomes zero, so the time required to reach the highest position is, from (1),

$$t_0 = \frac{v_0}{g \sin\alpha} \tag{3}$$

At that time, the displacement is

$$x_0 = \frac{1}{2}\frac{v_0^2}{g \sin\alpha} \tag{4}$$

For downward motion, the velocity and the displacement are described by

$$v = (g \sin\alpha)t \tag{5}$$

$$x = \frac{1}{2}(g \sin\alpha)t^2 \tag{6}$$

where we take a new origin for x and t at the highest position so that the initial conditions are $v(t = 0) = 0$ and $x(t = 0) = 0$.

We find the time required to move from the highest position to the starting position by substituting (4) into (6):

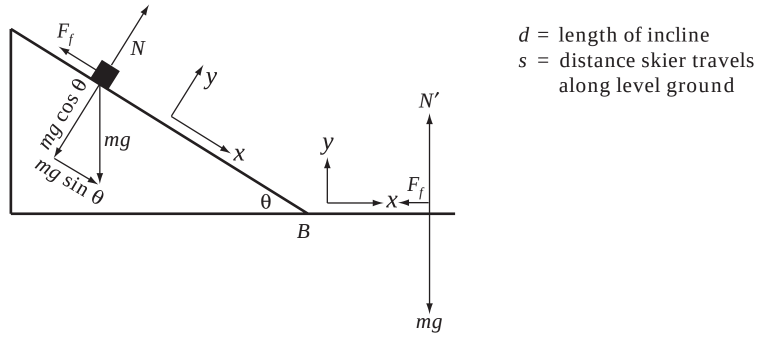
$$t' = \frac{v_0}{g \sin\alpha} \tag{7}$$

Adding (3) and (7), we find

$$\boxed{t = \frac{2v_0}{g \sin\alpha}} \tag{8}$$

for the total time required to return to the initial position.

2-24.



While on the plane:

$$\sum F_y = N - mg\cos\theta = m\ddot{y} = 0 \qquad \text{so } N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_f; \qquad F_f = \mu N = \mu mg\cos\theta$$

$$mg\sin\theta - \mu mg\cos\theta = m\ddot{x}$$

So the acceleration down the plane is:

$$a_1 = g(\sin\theta - \mu\cos\theta) = \text{ constant}$$

While on level ground: $N' = mg;\ F_f = -\mu mg$

So $\sum F_x = m\ddot{x}$ becomes $-\mu mg = m\ddot{x}$

The acceleration while on level ground is

$$a_2 = -\mu g = \text{ constant}$$

For motion with constant acceleration, we can get the velocity and position by simple integration:

$$\ddot{x} = a$$

$$v = \dot{x} = at + v_0 \tag{1}$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \tag{2}$$

Solving (1) for *t* and substituting into (2) gives:

$$\frac{v - v_0}{a} = t$$

$$x - x_0 = \frac{v_0(v - v_0)}{a} + \frac{1}{2}\cdot\frac{(v - v_0)^2}{a}$$

or

$$2a(x - x_0) = v^2 - v_0^2$$

Using this equation with the initial and final points being the top and bottom of the incline respectively, we get:

$$2a_1 d = V_B^2 \qquad V_B = \text{speed at bottom of incline}$$

Using the same equation for motion along the ground:

$$2a_2 s = -V_B^2 \tag{3}$$

Thus

$$a_1 d = -a_2 s \qquad a_1 = g(\sin\theta - \mu\cos\theta) \qquad a_2 = -\mu g$$

So

$$gd(\sin\theta - \mu\cos\theta) = \mu g s$$

Solving for μ gives

$$\mu = \frac{d\sin\theta}{d\cos\theta + s}$$

Substituting $\theta = 17°$, d = 100 m, s = 70 m gives

$$\boxed{\mu = 0.18}$$

Substituting this value into (3):

$$-2\mu g s = -V_B^2$$

$$V_B = \sqrt{2\mu g s}$$

$$\boxed{V_B = 15.6 \text{ m/sec}}$$

2-25.

a) At A, the forces on the ball are:

The track counters the gravitational force and provides centripetal acceleration

$$N - mg = mv^2/R$$

Get v by conservation of energy:

$$E_{top} = T_{top} + U_{top} = 0 + mgh$$

$$E_A = T_A + U_A = \frac{1}{2}mv^2 + 0$$

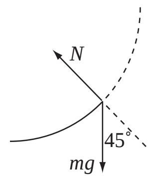
$$E_{top} = E_A \rightarrow v = \sqrt{2gh}$$

So

$$N = mg + m\,2gh/R$$

$$\boxed{N = mg\left(1 + \frac{2h}{R}\right)}$$

b) At B the forces are:

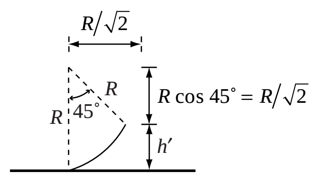


$$N = mv^2/R + mg\cos 45°$$

$$= mv^2/R + mg/\sqrt{2} \qquad (1)$$

Get v by conservation of energy. From a), $E_{total} = mgh$.

At B, $E = \frac{1}{2}mv^2 + mgh'$



$$R = \frac{R}{\sqrt{2}} + h' \quad \text{or} \quad h' = R\left(1 - \frac{1}{\sqrt{2}}\right)$$

So $E_{total} = T_B + U_B$ becomes:

$$mgh = mgR\left(1 - \frac{1}{\sqrt{2}}\right) + \frac{1}{2}mv^2$$

Solving for v^2

$$2\left[gh - gR\left(1 - \frac{1}{\sqrt{2}}\right)\right] = v^2$$

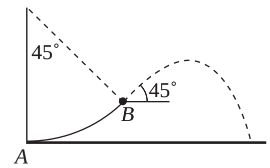
Substituting into (1):

$$N = mg\left[\frac{2h}{R} + \left(\frac{3}{\sqrt{2}} - 2\right)\right]$$

c) From b) $v_B^2 = 2g\left[h - R + R/\sqrt{2}\right]$

$$\boxed{v = \left[2g\left(h - R + R/\sqrt{2}\right)\right]^{1/2}}$$

d) This is a projectile motion problem



Put the origin at A.

The equations:

$$x = x_0 + v_{x0} t$$

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$$

become

$$x = \frac{R}{\sqrt{2}} + \frac{v_B}{\sqrt{2}} t \qquad (2)$$

$$y = h' + \frac{v_B}{\sqrt{2}} t - \frac{1}{2} g t^2 \qquad (3)$$

Solve (3) for t when $y = 0$ (ball lands).

$$g t^2 - \sqrt{2}\, v_B t - 2h' = 0$$

$$t = \frac{\sqrt{2}\, v_B \pm \sqrt{2 v_B^2 + 8 g h'}}{2g}$$

We discard the negative root since it gives a negative time. Substituting into (2):

$$x = \frac{R}{\sqrt{2}} + \frac{v_B}{\sqrt{2}} \left[\frac{\sqrt{2}\, v_B \pm \sqrt{2 v_B^2 + 8 g h'}}{2g} \right]$$

Using the previous expressions for v_B and h' yields

$$\boxed{x = \left(\sqrt{2} - 1\right) R + h + \left[h^2 - \frac{3}{2} R^2 + \sqrt{2}\, R^2 \right]^{1/2}}$$

e) $U(x) = m g y(x)$, with $y(0) = h$, so $U(x)$ has the shape of the track.