

Discrete Charge Densities

Cartesian

Numerous possible forms

Line of discrete charges (+x axis only)

$$\rho(\mathbf{r}) = \delta(y)\delta(z) \sum_i Q_i \delta(x - x_i)$$

Surface of discrete charges (xy plane)

$$\rho(\mathbf{r}) = \delta(z) \sum_i \sum_j Q_{ij} \delta(x - x_i) \delta(y - y_j)$$

Q_{ij} = the value of Q at point (x_i, y_j)

Volume of discrete charges

$$\rho(\mathbf{r}) = \sum_i \sum_j \sum_k Q_{ijk} \delta(x - x_i) \delta(y - y_j) \delta(z - z_k)$$

Q_{ijk} = the value of Q at point (x_i, y_j, z_k)

Spherical

Numerous possible forms

Line of discrete charges (+x axis only)

$$\rho(\mathbf{r}) = \left[\frac{\delta(\theta - \frac{\pi}{2})}{r} \right] \left[\frac{\delta(\varphi)}{r \sin \theta} \right] \sum_i Q_i \delta(r - r_i)$$

Surface of discrete charges (xy plane)

$$\rho(\mathbf{r}) = \left[\frac{\delta(\theta - \frac{\pi}{2})}{r} \right] \sum_i \sum_k Q_{ik} \delta(r - r_i) \left[\frac{\delta(\varphi - \varphi_k)}{r \sin \theta} \right]$$

Q_{ik} = the value of Q at point $(r_i, \pi/2, \varphi_k)$

Volume of discrete charges

$$\rho(\mathbf{r}) = \sum_i \sum_j \sum_k Q_{ijk} \delta(r - r_i) \left[\frac{\delta(\theta - \theta_j)}{r} \right] \left[\frac{\delta(\varphi - \varphi_k)}{r \sin \theta} \right]$$

Q_{ijk} = the value of Q at point $(r_i, \theta_j, \varphi_k)$

Cylindrical

Numerous
possible
forms

Line of discrete charges (*+x axis only*)

$$\rho(\mathbf{r}) = \delta(z) \left[\frac{\delta(\theta)}{\rho} \right] \sum_i Q_i \delta(\rho - \rho_i)$$

Surface of discrete charges (*xy plane*)

$$\rho(\mathbf{r}) = \delta(z) \sum_i \sum_j Q_{ij} \delta(\rho - \rho_i) \left[\frac{\delta(\theta - \theta_j)}{\rho} \right]$$

Q_{ik} = the value of Q at
point $(\rho_i, \theta_j, 0)$

Volume of discrete charges

$$\rho(\mathbf{r}) = \sum_i \sum_j \sum_k Q_{ijk} \delta(\rho - \rho_i) \left[\frac{\delta(\theta - \theta_j)}{\rho} \right] \delta(z - z_k)$$

Q_{ijk} = the value of Q at
point (ρ_i, θ_j, z_k)