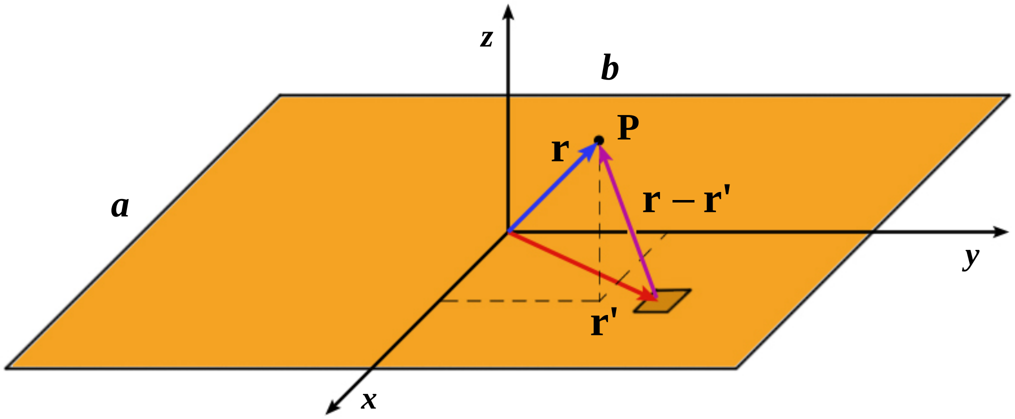
Find the Electric Field at point **P** due to a finite rectangular sheet that contains a uniform charge density σ.



$\mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$

$\mathbf{r'} = x'\,\hat{\mathbf{x}} + y'\,\hat{\mathbf{y}}$

$\mathbf{r} - \mathbf{r'} = \left(x - x'\right)\hat{\mathbf{x}} + \left(y - y'\right)\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$

For this problem, Cartesian coordinates would be the best choice in which to work the problem.

The electric field can be found using: $\mathbf{E} = \iint \frac{k_e \sigma dA}{\left|\mathbf{r}-\mathbf{r}'\right|^3}(\mathbf{r}-\mathbf{r}')$.

Since the sheet is in the *xy*-plane, the area element is $dA = dx'dy'$.

* Now we need expressions for & in terms of space (x, y, z) and body (x', y', z') coordinates.

$$\mathbf{r} - \mathbf{r}' = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$

$$\left|\mathbf{r} - \mathbf{r}'\right| = \sqrt{(x-x')^2 + (y-y')^2 + z^2}$$

$$\left|\mathbf{r} - \mathbf{r}'\right|^3 = \left((x-x')^2 + (y-y')^2 + z^2\right)^{3/2}$$

∴

$$\mathbf{E} = \iint \frac{k_e \sigma dx'dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}(x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$

This integral can be easier to deal with if it is broken down into component form:

$$\mathbf{E} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} + E_z\hat{\mathbf{z}}$$

$\therefore$

$$E_x = k_e\sigma \iint \frac{(x-x')dx'dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}$$

$$E_y = k_e\sigma \iint \frac{(y-y')dx'dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}$$

$$E_x = k_e\sigma \iint \frac{z\,dx'dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}$$

The limits on *dx'* and *dy'* are those that define the dimensions of the sheet:

$$-\frac{a}{2} \le dx' \le \frac{a}{2}$$
$$-\frac{b}{2} \le dy' \le \frac{b}{2}$$

$\therefore$

$$E_x = k_e\sigma \int_{-\frac{a}{2}}^{\frac{a}{2}} (x-x')dx' \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}$$

$$E_y = k_e\sigma \int_{-\frac{a}{2}}^{\frac{a}{2}} dx' \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{(y-y')dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}$$

$$E_z = k_e\sigma z \int_{-\frac{a}{2}}^{\frac{a}{2}} dx' \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{dy'}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}$$

Note: *z* can be taken in front of both integrals since it does not depend on *x'* or *y'*.

* The *dx'* integral can **NOT** be performed until the *dy'* integral is evaluated since there is *x'* dependence in the *dy'* integral.

Comments:

* The analytical or closed form solution is extremely long and nasty for points off the *z*-axis and will not be shown here.

* If **P** is at some point on the *z*-axis only, the E_x and E_y components vanish (*due to symmetry*) and only the E_z component is left to evaluate (*the solution is still somewhat nasty*).