Math Physics: HWK #2

1) Find the dot product for the following vectors:

a) $\mathbf{v} = \langle 3,4 \rangle$ & $\mathbf{u} = \langle 2,-3 \rangle$

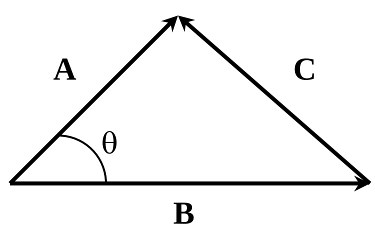
b) $\mathbf{v} = \langle 4,-2 \rangle$ & $\mathbf{u} = \langle 1,-1 \rangle$

c) $\mathbf{v} = \langle 1,0,2 \rangle$ & $\mathbf{u} = \langle 2,-3,5 \rangle$

d) $\mathbf{v} = \langle 6,7,-2 \rangle$ & $\mathbf{u} = \langle 9,1,0 \rangle$

2) Find the angle between each vector pair in problem 2.

3) Let **C** = **A** – **B** (*show in figure below*)



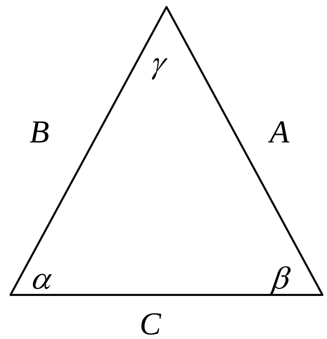
Find $\mathbf{C}\cdot\mathbf{C}$ [*using* $\mathbf{C}\cdot\mathbf{C} = (\mathbf{A}-\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})$] in terms of magnitudes (*A*, *B* & *C*) and θ only.

** By what familiar name does this reduced expression go by?

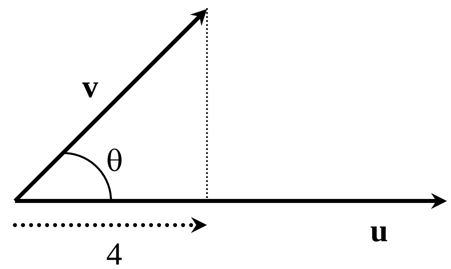
(**Hint**: Look in any trig book under *Solution of Oblique Triangles*)

4) Use the vector cross product to express the area of a triangle in three different ways.
Hence, prove the Law of Sines:

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$



5) Given $\mathbf{u} = \langle 10, 0 \rangle$



a) Find **v** and θ such that its projection onto **u** is equal to 4.

b) Is your answer for **v** and θ in part *a*) unique? Why or why not?

6) Find the cross product for the following pair of vectors:

a) $\langle 1, -4, 1 \rangle$ & $\langle 2, 3, 0 \rangle$

b) $\langle 3, 2, -1 \rangle$ & $\langle 5, 2, 3 \rangle$

c) $\langle 1, 2, 3 \rangle$ & $\langle 1, 2, 3 \rangle$

7) Let $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ & $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$. Verify the following cross-product identities:

a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$

b) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$ **Note**: $\mathbf{0} = \langle 0, 0, 0 \rangle$

8) Let $\mathbf{A} = \langle a_1, a_2, a_3 \rangle$ & $\mathbf{B} = \langle b_1, b_2, b_3 \rangle$. Prove that $|\mathbf{A} \times \mathbf{B}| = AB \sin\theta$ by evaluating both sides and showing they are equal.

Hint: Use the trig. identity $\sin\theta = \sqrt{1 - \cos^2\theta}$ and the expression for $\cos\theta$ from the dot product to show that both sides reduce to the same expression.

9) Find and evaluate all possible cross-product combinations for the unit vectors in Cartesian coordinates.

Hint: There will be 9 of them $\left(\hat{\mathbf{e}}_i \times \hat{\mathbf{e}}_j \right)$.

10) Calculate the Area of a parallelogram with the following vectors as adjacent sides:

$$\langle 2,-1,0\rangle \quad \& \quad \langle -1,2,0\rangle$$

11) Calculate the Volume of a parallelepiped with the following vectors as adjacent sides:

$$\langle 1,3,1\rangle \qquad \langle 0,6,6\rangle \qquad \langle -4,0,-4\rangle$$

12) A particle moves under the influence of electric and magnetic fields **E** and **B**. Show that a particle moving with an initial velocity

$$\mathbf{v_o} = \left(\frac{1}{B^2}\right)\mathbf{E}\times\mathbf{B}$$

is not accelerated (**a** = 0) if **E** is perpendicular to **B**.

Hint: Substitute $\mathbf{v_o}$ in to the Lorentz Force, $\mathbf{F} = q\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)$, and then combine with $\mathbf{F} = m\mathbf{a}$ to get an expression for **a**. The following identity will be most helpful:

$$(\mathbf{A}\times\mathbf{B})\times\mathbf{C} = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{A}(\mathbf{B}\cdot\mathbf{C})$$

13) Points P and P' have Cartesian coordinates (x, y, z) and (x', y', z'), cylindrical coordinates (ρ, φ, z) and (ρ', φ', z'), and spherical coordinates (r, θ, φ) and (r', θ', φ'), respectively. Write $|\,\mathbf{r}-\mathbf{r}'\,|$ & $|\,\mathbf{r}\,|$ in all three coordinate systems.

Hint: Use the relation $|\mathbf{a}| = \sqrt{\mathbf{a}\cdot\mathbf{a}}$, with $\mathbf{a} = \mathbf{r}-\mathbf{r}'$ and **r** and **r'** written in terms of appropriate unit vectors.