HWK #3

Name ________________________

Show all work. You may use Mathcad or some other software to check your answer.

1. Find the volume of a sphere of radius R by integrating over the primary volume elements in Cartesian coordinates. (*Be mindful of your limits of integration*)

   Hint: Use a trig substitution for your integral over dy.

2. Express $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ as partial derivatives in spherical coordinates using the chain rule.

3. Express $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ as partial derivatives in cylindrical coordinates using the chain rule.

4. Find $\frac{\partial \hat{\theta}}{\partial r}$, $\frac{\partial \hat{\theta}}{\partial \theta}$, $\frac{\partial \hat{\theta}}{\partial \varphi}$ and $\frac{\partial \hat{\varphi}}{\partial r}$, $\frac{\partial \hat{\varphi}}{\partial \theta}$, $\frac{\partial \hat{\varphi}}{\partial \varphi}$ in terms of other spherical unit vectors.

5. Find the partial derivates $\frac{\partial}{\partial \rho}$, $\frac{\partial}{\partial \varphi}$ and $\frac{\partial}{\partial z}$ for all 3 cylindrical unit vectors in terms of other cylindrical unit vectors.

6. Show that $f(x,t) = x \pm ct$ satisfies the one dimensional **Wave Equation**:

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

7. Find the partial derivatives of the following functions at the given points with respect to the given variables:

$f = e^{xyz}$ with respect to x @ (1,0,-1)

$f = \cos\left(\frac{xy}{z}\right)$ with respect to z @ (π,1,1)

$f = x^2 y + y^2 z + z^2 x$ with respect to y @ (1,-1,2)

$f = \sqrt{x^2 + y^2 + z^2}$ with respect to x @ (x, y, z)

8. Given that $f'' - \alpha f = 0$, $g'' - \beta g = 0$ and $h'' - \gamma h = 0$, write an equation relating α, β, and γ such that

$$F(x, y, z) = f(x)g(y)h(z)$$

satisfies the three-dimensional **Laplace's Equation**:

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 0$$

Note: f'' means $\frac{\partial^2 f}{\partial x_i^2}$ *or* $\frac{\partial}{\partial x_i}\left(\frac{\partial f}{\partial x_i}\right)$, where x_i is some coordinate variable.