Show all work. You may use Mathcad or some other software to check your answer.

1. Show that
\[ \vec{\nabla} \times (\vec{\nabla} \Phi) = 0 \]
for any scalar field \( \Phi \).

2. Show that
\[ \vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0 \]
for any vector field \( \vec{F} \).

3. Calculate the divergence and the curl of each of the following vector fields:
   
   a) \( \vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \)
   
   b) \( \vec{V} = x^2\hat{x} + y^2\hat{y} + xyz\hat{z} \)
   
   c) \( \vec{V} = x\sin y\hat{x} + \cos y\hat{y} + xy\hat{z} \)

4. Evaluate the integral
\[ \oint_C \vec{u} \cdot d\vec{l} \]
by converting it to an area integral using Stoke’s Theorem if:
   
   a) \( C \) is a unit circle with ccw orientation in the \( x-y \) plane, centered at the origin and
   \[ \vec{u} = x^2y\hat{x} - xy^2\hat{y} \]

   b) \( C \) is a semicircle of radius \( a \) in the \( x-y \) plane with ccw orientation with the flat side along the \( x \)-axis, the center of the circle at the origin and
   \[ \vec{u} = xy^2\hat{x} + x^2y\hat{y} \]

   c) \( C \) is a 3-4-5 right triangle with ccw orientation with the sides of length 3 and 4 along the \( x- \) and \( y- \)axes, respectively and
   \[ \vec{u} = x^2\hat{x} + xy\hat{y} \]
5. Evaluate the integral
\[ \int_S \mathbf{v} \cdot d\mathbf{A} \]
by converting it to a volume integral using the Divergence Theorem if:

a) \( S \) is a sphere of radius 2 centered on the origin and
\[ \mathbf{v} = x^3 \hat{x} + 3yz^2 \hat{y} + 3y^2z\hat{z} \]

b) \( S \) is a hemisphere of radius 1, centered at the origin, with the flat side in the \( x-y \) plane and
\[ \mathbf{v} = x^2yz(\hat{y} + \hat{z}) \]

6. Show that the vector
\[ \mathbf{u} = x\hat{x} + y\hat{y} - 2z\hat{z} \]
has zero divergence and a zero curl.

7. Derive the expression for the gradient operator in spherical coordinates. Then use this result to obtain expressions for the divergence and curl in spherical coordinates.

**Hint I:** For the divergence and curl, use an arbitrary function \( F(r, \theta, \phi) \) to help in the proof

**Hint II:** For the curl, use the relations
\[ (a + b) \times c = a \times c + b \times c \quad \text{and} \quad \hat{x}_r \frac{\partial}{\partial q} \times F = \hat{x}_r \times \frac{\partial F}{\partial q} \]