HWK #5

Name ________________________

Show all work. You may use Mathcad or some other software to check your answer.

1. Evaluate the following integrals:

   a) $\int_0^5 (\sin x)\delta(x+1)\,dx$

   b) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(x^2 y + xy^2 + 1\right)\delta(x)\delta(y-1)\,dx\,dy$

   c) $\int_0^{\pi} \left(\cos 2\theta\right)\delta'\left(\theta - \frac{\pi}{2}\right)d\theta$

2. Show that $\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r_o}|}\right) = -4\pi\delta(\mathbf{r}-\mathbf{r_o})$ using the result $\vec{\nabla}\cdot\left(\frac{\mathbf{r}-\mathbf{r_o}}{|\mathbf{r}-\mathbf{r_o}|^3}\right) = 4\pi\delta(\mathbf{r}-\mathbf{r_o})$.

3. Referring to the example worked in class for an infinite line of alternating charges, find the electric field **E**(*x*,*y*,*z*) at any point P.

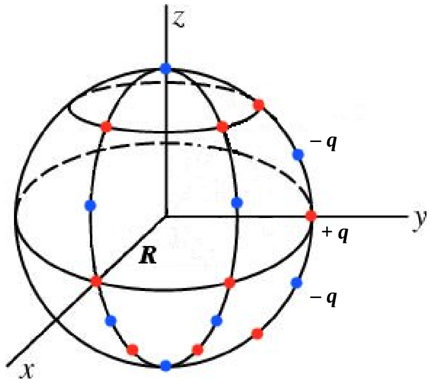
   NOTE: For this problem, $\mathbf{r} - \mathbf{r}' = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}$

4. Find **E** for an infinite line of alternating charges only along the *x*-axis if

   $$\phi = k_e q\left(\frac{1}{x} + 2\sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt{x^2 + k^2 a^2}}\right)$$

   NOTE: Your answer will look like our expression for E_x that we found in the notes or from prob. 10 after setting $y = z = 0$.

5. Find $\phi(r)$ along the + z-axis for a set of alternating charges symmetrically arrayed on the surface of a sphere as shown in the following figure.



In the θ direction, charges are arrayed every 30° from 0 to π.

NOTE: There are 8 charges on each ring at every fixed θ except at the poles

In the φ direction, charges are arrayed every 45° from 0 to 2π.

Hint: Treat the top and bottom charges separate from the rings

➔ $\rho_{tot} = \rho_{top} + \rho_{bottom} + \rho_{rings}$

* Graph $\dfrac{\phi}{k_e q}$ for z from 0 to 10 using R = 5.

6. Find **E** for the previous problem along the + z-axis using: $\mathbf{E} = -\vec{\nabla}\phi$.

* Graph $\dfrac{\mathbf{E}}{k_e q}$ for z from 0 to 10 using R = 5. What is **E**(0)? Does it equal what you would expect for this type of charge distribution? Why or why not?

7. In Cartesian coordinates, the Laplacian operator (∇^2) is given by:

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Prove that in Cylindrical coordinates, the Lapacian operator (∇^2) is given by:

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2}{\partial \varphi^2}\right) + \frac{\partial^2}{\partial z^2}$$

NOTE: To put your answer in this form, use the relation:

$$\frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \rho^2} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right)$$