Show all work. You may use Mathcad or some other software to check your answer.

1. Evaluate the following integrals:
   
   a) \[ \int_0^5 (\sin x)\delta(x + 1)\,dx \]

   b) \[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( x^2y + xy^2 + 1 \right)\delta(x)\delta(y - 1)\,dxdy \]

   c) \[ \int_0^{\pi} \left( \cos 2\theta \right)\delta\left( \theta - \frac{\pi}{2} \right)\,d\theta \]

2. Show that \[ \nabla^2 \left( \frac{1}{|\mathbf{r} - \mathbf{r}_o|} \right) = -4\pi \delta(\mathbf{r} - \mathbf{r}_o) \] using the result \[ \nabla \cdot \left( \frac{1}{|\mathbf{r} - \mathbf{r}_o|} \right) = 4\pi \delta(\mathbf{r} - \mathbf{r}_o). \]

3. Referring to the example worked in class for an infinite line of alternating charges, find the electric field \( \mathbf{E}(x,y,z) \) at any point \( P \).
   
   **NOTE:** For this problem, \( \mathbf{r} - \mathbf{r}' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z} \)

4. Find \( \mathbf{E} \) for an infinite line of alternating charges only along the \( x \)-axis if
   
   \[ \phi = kq \left( \frac{1}{x} + 2\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{x^2 + k^2a^2}} \right) \]

   **NOTE:** Your answer will look like our expression for \( E_x \) that we found in the notes or from prob. 10 after setting \( y = z = 0. \)
5. Find $\phi(r)$ along the $+z$-axis for a set of alternating charges symmetrically arrayed on the surface of a sphere as shown in the following figure.

* Graph $\frac{\phi}{k_\epsilon q}$ for $z$ from 0 to 10 using $R = 5$.

6. Find $E$ for the previous problem along the $+z$-axis using: $E = -\nabla \phi$.

* Graph $\frac{E}{k_\epsilon q}$ for $z$ from 0 to 10 using $R = 5$. What is $E(0)$? Does it equal what you would expect for this type of charge distribution? Why or why not?

7. In Cartesian coordinates, the Laplacian operator ($\nabla^2$) is given by:

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Prove that in Cylindrical coordinates, the Laplacian operator ($\nabla^2$) is given by:

$$\nabla^2 = \frac{1}{\rho \partial \rho} \left( \rho \partial \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \left( \frac{\partial^2}{\partial \phi^2} \right) + \frac{\partial^2}{\partial z^2}$$

**NOTE:** To put your answer in this form, use the relation:

$$\frac{1}{\rho \partial \rho} \frac{\partial^2}{\partial \rho^2} = \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right)$$