

HWK #6

Name _____

Show all work. You may use Mathcad or some other software to check your answer.

1. Let $z = 3 + 4i$ & $w = 2 - i$, find

a) $z + w$

b) $z - w$

c) $z \times w$

d) z / w

e) $|z + w|$

f) z & w in exponential polar form $[re^{i\theta}]$

2. Find z in Cartesian form given the following polar form information:

a) $|z| = 7$, $\text{Arg } z = \frac{25\pi}{36}$

b) $|z| = 2$, $\text{Arg } z = -\frac{\pi}{6}$

c) $e^{\frac{i\pi}{2}}$

d) $e^{1-i\pi}$

3. Show that

$$\sin^2 \theta + \cos^2 \theta = 1$$

using the complex forms for $\sin\theta$ and $\cos\theta$.

4. Prove the reciprocal identity using the complex exponential polar form for z .

$$z z^{-1} = 1$$

5. Find the all of the roots (*real and complex*) for the following:

a) $(-i)^{\frac{1}{3}}$

b) $(-1)^{\frac{1}{3}}$

c) $(i)^{\frac{1}{3}}$

6. Find all the roots of $(-2 + 2i)^{\frac{1}{3}}$.
7. Find the value of z for which $z^{\frac{1}{5}} = 1$.
8. Find all the values of z for which $z^5 = -1$ and graph these points on the complex plane.
9. List the next 8 terms for the repeating series $i^n = i, -1, -i, 1, \dots$ and determine a power law to predict the next repeating term for each value.
10. Evaluate the following expressions:
- $\ln(i)$
 - $\text{Ln}(i)$
 - $(1-i)^{3i+1}$ for all θ
 - $(-1+3i)^{-2i}$ for $-\pi < \theta < \pi$
 - $^{-1+3i}\sqrt[3]{(2+i)}$ for all θ
 - $(1+5i)^{\frac{1}{-4i}}$ for $-\pi < \theta < \pi$
11. Show that $e^{\pm 2\pi i n} = 1$.
12. Prove De Moivre's theorem:
 $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$