HWK #7

Name ________________________

Show all work. You may use Mathcad or some other software to check your answer.

1. Show that

   $f(z) = z$ is analytic

   but

   $f(z) = z^*$ is not.

2. Show that the analytic function $w(z) = u + iv$ satisfies $\nabla^2 w = 0$ in 2D. Such functions are called **Harmonic functions**.

   Hint: show that $\nabla^2 u = 0$ & $\nabla^2 v = 0$.

3. Determine if the following functions are harmonic (*see Prob*. 2):

   a) $u(x, y) = x^2 + y^2$

   b) $u(x, y) = x^2 - y^2$

4. Determine the region where each function is analytic:

   a) y^2

   b) $\dfrac{z^2 + z}{z(z^2 + 1)}$

   c) $e^{x^2 - y^2}(\cos 2xy + i \sin 2xy)$

5. Determine *all* Taylor or Laurent series for the following functions about the point specified:

   a) $ln(1 + z)$ about $z = 0$

   b) $\dfrac{1}{z^2 + 1}$ about $z = i$

6. Determine the location and type of singularity(*ies*) that exists for the following functions:

a) $\dfrac{1}{z^2+9}$

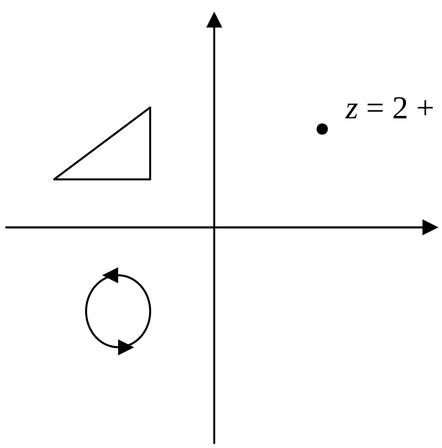
b) $\dfrac{\cos z}{z} - \dfrac{\sin z}{z^2}$

7. Let $f(z) = u(x,y) + iv(x,y)$ be differentiable at a non-zero point. Show that the polar from of $f(z)$ at this point satisfies the **Polar form of the Cauchy-Riemann Equations**:

$$r \cdot u_r = v_\theta$$
$$r \cdot v_r = -u_\theta$$

8. Map the following points in the *z*-plane into the *w*-plane under the following function

$w = z^*$



9. Approximate the following using the 1st 2 terms of a binomial series expansion.

$(1+2x^2)^3 \quad at \quad x = 0.03$

a) What is the percent error compared to the actual value?

b) How many expansion terms would be necessary to get 6 decimal places of accuracy?
Hint: Use Mathcad

10. Approximate the following function using the 1st 4 terms of its series expansion to 8 decimal places.

$\cos(x) \quad at \quad x = \pi$

a) What is the percent error compared to the actual value?

b) How many expansion terms would be necessary to be less than 0.001% accurate?
Hint: Use Mathcad

11. Evaluate the following integrals:

$a)\ \oint_C \frac{\cos z}{z} dz$, where C is a circle of radius 2 centered at the origin

$b)\ \oint_C \frac{z-1}{z+2} dz$, where C is a circle of radius 1 centered at the origin

12. Evaluate the following integrals:

$a)\ \int_0^{2\pi} \frac{1+\cos\theta}{2-\sin\theta} d\theta$

$b)\ \int_0^{2\pi} \frac{1}{1+\sin^2\theta} d\theta$

13. Evaluate the following integrals:

$a)\ \int_{-\infty}^{+\infty} \frac{1}{x^2+2} dx$

$b)\ \int_{-\infty}^{+\infty} \frac{x^2}{(1+x^2)^2} dx$