HWK #8

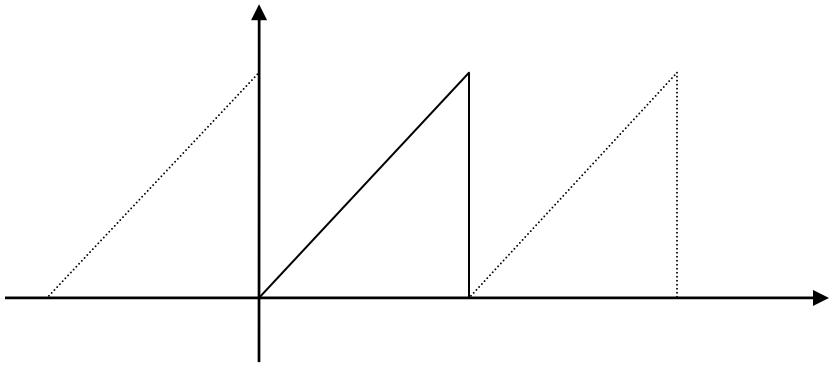
Name _____

Show all work. You may use Mathcad or some other software to check your answer.

1. Show that $f(x+L) = f(x)$ for the Fourier series

$$f(x) = a_o + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2n\pi x}{L}\right) + b_n \sin\left(\frac{2n\pi x}{L}\right)\right)$$

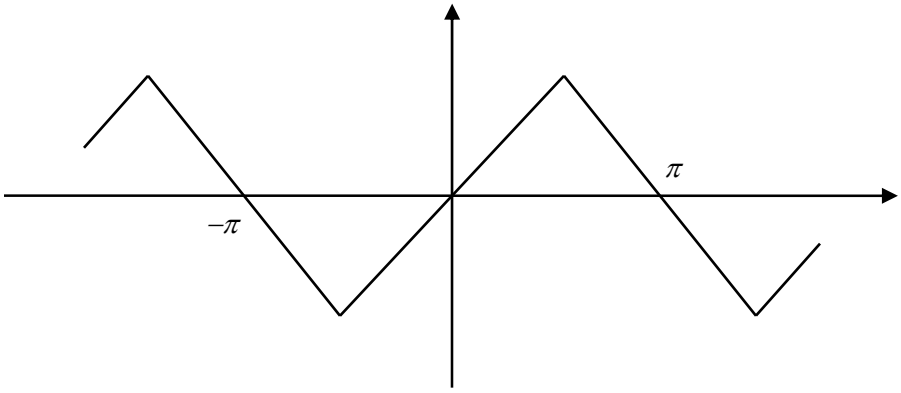
2. Sawtooth Potential



$$V(t) = V_o \frac{t}{T} \qquad 0 \le t \le T$$

Find the coefficients a_o, a_n & b_n for the Fourier series representation of this function. Plot your solution for several values of n (1, 10, 50) if V_o = 10 V and T = π *sec* from 0 to 2π..

3. The *Zig-Zag* Function



$$f(x) = \begin{cases} x & for \ -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ \pi - x & for \ \frac{\pi}{2} \le x \le \frac{3\pi}{2} \end{cases}$$

Find the coefficients a_o, a_n & b_n for the Fourier series representation of this function. Simplify as much as possible (*i.e. redefine indices to remove any regularly occurring terms in the summation that are zero*). Plot your solution for several values of n (0, 2, 100) from – π to + π.

4. Find the Fourier Transform [F(k)] of the following functions:

a) $f(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

b) $f(x) = \begin{cases} xe^{-\alpha x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$

c) $f(x) = \dfrac{x}{x^2 + a^2}$ ➔ *Do not try to combine answers.*

5. Find the Laplace Transform for each of the following functions:

a) $f(t) = \sinh(\alpha t)$

b) $f(t) = t\cosh(\alpha t)$

c) The Sawtooth function:

$f(t) = a(t - nT)$ *if* $nT < t < (n+1)T$

Hint: Let $g(t) = at$