HWK #9

Name _____

Waves on a finite and infinite String

Show all work. You may use Mathcad or some other software to check your answer.

1. Consider a finite string bound between 0 and L. At t = 0, the initial conditions are:

a) $y(x,0) = x(L-x)$ **B.C.:** $y(0,t) = y(L,t) = 0$

$v_o(x) = 0$

** Find $y(x,t)$ and plot $y(x,0)$ using N = 400 and L = 5 *cm* on the same graph with $y_n(x,0)$ for n values of 1, 3, 5 and 7.

b) $y(x,0) = 0$ **B.C.:** $y(0,t) = y(L,t) = 0$

$$v_o(x) = \begin{cases} -1 & for \ \ |x-d| \le a \\ 0 & for \ \ 0 \le x \le d-a \ \ and \ \ d+a \le x \le L \end{cases}$$

Note: The I.C. in part (*b*) correspond to a string struck by a hammer of width 2*a* and unit velocity at the point $x = d$.

** Find $y(x,t)$ and plot $y(x,t)$ for the following values of t: 0, 0.01, 0.02, 0.03, 0.05 on the same graph with N = 400, L = 20 *cm*, v = 80 *cm/s*, d = 10 *cm* and a = 2.5 *cm*.

2. Consider a finite string bound between 0 and L. At t = 0, the initial conditions are:

$$y(x,0) = \begin{cases} 0 & for \ \ 0 < x \le x_o \ \ and \ \ x \ge L \\ A & for \ \ x_o < x < L \end{cases}$$ **B.C.:** $y(0,t) = y(L,t) = 0$

$v_o(x) = 0$

NOTE: This is the profile for a **step function** at x_o.

** Find $y(x,t)$. Using A = 0.5, L = 20 and x_o = 10, plot the initial wave profile $y_o(x)$ on one graph with N = 2100 and the 1st five non-zero modes of $y_n(x,0)$ all on a different graph.