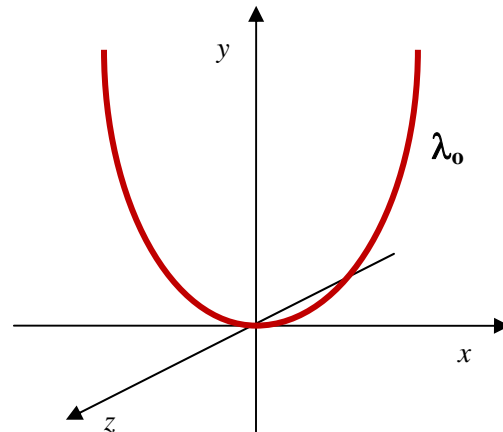


Representing the density function of a linear distribution of mass using a delta function



The wire is in the shape of a parabola given by:

$$y = Cx^2 \quad w/ z = 0 \text{ everywhere}$$

Let $\rho_m(x, y, z) = k\lambda_0\delta(y - Cx^2)\delta(z)$ model the above linear mass distribution.

If this charge density is correct, the following **MUST** be true:

$$\int_L \lambda_m(\mathbf{r}) ds = \int_V \rho_m(\mathbf{r}) dV \quad \text{which comes from:} \quad \lambda = \frac{dm}{dl} \quad , \quad \rho = \frac{dm}{dV}$$

(total mass = total mass)

$$\lambda dl = dm \quad , \quad \rho dV = dm$$

$$\rightarrow \lambda dl = \rho dV$$

Use this equality to find an exact expression for k .

In Cartesian coordinates:

$$ds = \sqrt{dx^2 + dy^2 + dz^2}$$

With $z = 0$ and $y = Cx^2$

$$\rightarrow dz = 0 \quad \& \quad dy = 2Cxdx$$

\therefore

$$ds = \sqrt{1 + 4C^2x^2} dx$$

Combining and expanding, the equality becomes:

$$\int \lambda_o \sqrt{1+4C^2 x^2} dx = \iiint k \lambda_o \delta(y - Cx^2) \delta(z) dx dy dz$$

But

$$\int_{-\infty}^{\infty} \delta(z) dz = 1$$

$$\rightarrow \int \lambda_o \sqrt{1+4C^2 x^2} dx = \iint k \lambda_o \delta(y - Cx^2) dx dy$$

But

$$\int_{-\infty}^{\infty} \delta(y - Cx^2) dy = 1 \quad \text{since } x \text{ is held constant during the integral.}$$

$$\rightarrow \int \lambda_o \sqrt{1+4C^2 x^2} dx = \int k \lambda_o dx$$

\therefore

$$k = \sqrt{1+4C^2 x^2}$$

The full mass density function is then:

$$\rho_m(x, y, z) = \lambda_o \sqrt{1+4C^2 x^2} \delta(y - Cx^2) \delta(z)$$