

## Finding the mass of a segment of a spring

Assume the linear mass density of the spring is given by:  $\lambda = 1 + z$

The coil of the spring can be described using the function:  $\mathbf{r}(t) = \frac{1}{\sqrt{2}} \langle \cos t, \sin t, t \rangle \text{ cm}$



Find the mass of segment of spring that contains three full windings (red region).

→  $t = [0, 6\pi]$

Since  $\lambda = \frac{dm}{dl}$ , this implies that the total mass along  $dl$  is:

$$\begin{aligned} \text{Mass} &= \int \lambda dl \\ &= \int_0^{6\pi} (1+z) \sqrt{\dot{f}^2 + \dot{g}^2 + \dot{h}^2} dt \\ &= \int_0^{6\pi} \left(1 + \frac{t}{\sqrt{2}}\right) \sqrt{\frac{1}{2}(\sin^2 t + \cos^2 t) + \frac{1}{2}} dt \\ &= \int_0^{6\pi} \left(1 + \frac{t}{\sqrt{2}}\right) dt \end{aligned}$$

$\approx 144.47 \text{ g}$

$$\begin{aligned} dl &= |d\mathbf{r}| \\ \dot{f} &= \frac{-\sin t}{\sqrt{2}} \\ \dot{g} &= \frac{\cos t}{\sqrt{2}} \\ \dot{h} &= \frac{1}{\sqrt{2}} \end{aligned}$$