Modeling Uniform Motion

We believe that mere movement is life, and that the more velocity it has, the more it expresses vitality.

-- Rabindranath Tagore

The Tortoise and the Hare

Race is along a straight 2 mile track

Tortoise (0.2 *mph*)

Runs at a constant rate

Rabbit (15 *mph*)

Runs for 5 min
Plays and sleeps for 9 hr 53 min
Finishes race in 2 min

How far ahead of the rabbit is the tortoise when he crosses the finish line?

1 mile
.75 mile
.5 mile
.25 mile

Linear (Straight Line) Motion – Model #1

The simplest motion model is defined as the total ***distance*** traveled divided by the elapsed transit time. This quantity is called the **average speed** of an object.

$$\rightarrow \quad \textit{rate of linear motion} = \frac{\text{distance}}{\text{elapsed time}} \equiv \textbf{average speed}$$

The mathematical form of the average speed model is:

$v_{avg} \equiv \frac{d}{\Delta t}$

Δ means "change in"

$\Rightarrow \quad \Delta t = t_{final} - t_{initial}$

By ***choosing*** $t_{initial}$ to be zero at the start of **all** time measurements, this expression simplifies to:

$$v_{avg} \equiv \frac{d}{t}$$

The SI units associated with speed are:

$$\frac{\text{distance}}{\text{elapsed time}} = \frac{length}{time} = \frac{m}{s}$$

Speed is <u>always</u> a positive value because distance and time are always positive!

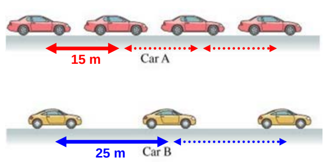
NOTE: *'**fast**' and '**slow**' are relative terms used primarily when comparing one objects rate of motion to another.*

Ex.

A bird travels <u>faster</u> than a snail, but <u>slower</u> than a bullet.

Ex.

Previously, we determined that car **B** was traveling "faster" than car **A**. Assuming that the distances shown below were both measured during 2 *s* time intervals, find the average speed of each car to support or discount our previous conclusion.



A

$$v_{avg} = \frac{15\,m}{2\,s} = 7.5\,\tfrac{m}{s}$$

B

$$v_{avg} = \frac{25\,m}{2\,s} = 12.5\,\tfrac{m}{s}$$

B is traveling "faster".

Ex.

A car travels at a constant speed of 55 *mph*. How far will it travel in 3 *hrs*?

NOTE: If the speed is constant, then $v_{constant} = v_{avg}$

$$d = v_{avg} t$$

$$d = \left(55\frac{mi}{hr}\right)(3hr)$$

$$d = 165\,mi$$

Ex.

If you travel from Arkadelphia to Little Rock in ¾ *hr* (a distance of ~60 *mi*), what is your average speed in ***m/s***?

Method I

Converting Units at the End

$$v_{avg} = \frac{d}{t}$$

$$v_{avg} = \frac{60\,mi}{.75\,hr} = 80\,mph$$

$$(80\,mph)\left(\frac{.447\frac{m}{s}}{1\,mph}\right) = 35.76\frac{m}{s}$$

Method II

Converting Units Upfront

$$d = (60\,mi)\left(\frac{1609\,m}{1\,mi}\right) = 96540\,m$$

$$t = (.75\,hr)\left(\frac{3600\,s}{1\,hr}\right) = 2700\,s$$

$$v_{avg} = \frac{96540\,m}{2700\,s} = 35.76\frac{m}{s}$$

The Tortoise and the Hare (*Continued*)

- Time it took Tortoise to complete the race:

$$t = \frac{d}{v} = \frac{2\ miles}{.2\ mph}$$

$$t = 10\,hr$$ Time required to cover a distance of 2 miles

- Distance the rabbit covers in 10 hrs:

$$d = (15\,mph)(5\min) + (0\,mph)(9\,hr\,53\min) + (15\,mph)(2\min)$$

$$d = (15\,mph)\left(\frac{1}{12}hr\right) + (15\,mph)\left(\frac{1}{30}hr\right)$$

$$d = 1.75\,miles$$ **The rabbit is a quarter mile (0.25) behind!**

Linear (Straight Line) Motion – Model #2

Average velocity

$$\textit{directional rate of linear motion} = \frac{\text{net displacement}}{\text{elapsed time}} \equiv \textbf{average velocity}$$

The mathematical form of our new velocity model is:

$$\vec{v}_{avg} \equiv \frac{\Delta\vec{x}}{\Delta t} \quad or \quad \mathbf{v}_{avg} \equiv \frac{\Delta \mathbf{x}}{\Delta t}$$

The SI units that are associated with velocity are:

$$\frac{\text{displacement}}{\text{elapsed time}} = \frac{length}{time} = \frac{m}{s}$$

Warning:

The **units** for **speed** and **velocity** are exactly the same. This results from both models measuring a *change in position* (length) *divided by time*.

Unlike *Speed* which is always a positive, *velocity* can be negative **if** the displacement is in a negative direction [due to the choice of coordinate system].

Since velocity is a vector quantity, we can represent it using vector arrows:

- direction of the motion = direction of the arrow
- magnitude of the velocity = length of the arrow.

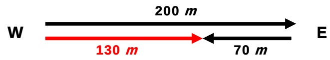
Warning:

To specify a *velocity* numerically, a number **AND** a direction must be given. If no direction is given, the value could easily be misinterpreted as *speed*.

Distinguishing Between Speed & Velocity

Ex. A car travels East 200 *m* in 8 *s* and then returns due West 70 *m* in 5 *s*. What is the *average speed* and *average velocity* of the car?

Visual Model of the Event



Speed:

$d = \bar{v}t$

$d = 200\,m + 70\,m = 270\,m$

$t = 8\,s + 5\,s$

$\bar{v} = \frac{270m}{13s} = 20.8\frac{m}{s}$ (~ 46.5 mph)

Velocity (in 1 D):

$(\mathbf{v}_x)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{200\,m - 70\,m}{13\,s - 0\,s}$

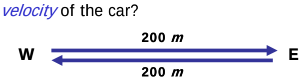
$(\mathbf{v}_x)_{avg} = \frac{130m}{13s} = 10\frac{m}{s}$ **East** (~ 24.4 mph)

DEMO – Around the Room

- In order to meet at the same place at the same time, the person with the greater distance to cover **MUST** travel at a higher rate of motion (speed)!

Ex. A car makes a round trip of 400 m (East 200 *m* then West 200 *m*) in 16 *s*. What is the *average speed* and *average velocity* of the car?

Visual Model



Speed:

$\bar{v} = \frac{400m}{16s} = 25\frac{m}{s}$ (~ 60 mph)

Velocity:

$(\mathbf{v}_x)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{0m}{16s} = 0\frac{m}{s}$

Ex. A car travels East 200 *m* in 8 *s*. What is the *average speed* and *average velocity* of the car?

Visual Model **W** —— **200 *m*** ——→ **E**

Speed:

$\bar{v} = \frac{200m}{8s} = 25\frac{m}{s}$ **(~ 60 mph)**

Velocity:

$(\mathbf{v}_x)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{200m}{8s} = 25\frac{m}{s}$ **East** **(~ 60 mph)**

Speed and velocity can ONLY have the same magnitude if the *distance* and *displacement* have the same magnitude.

- Real world example in the difference between speed and velocity

Physics Today
Sept. 2003
Pgs. 13-14

Linear (Straight Line) Motion – Model #3

Average acceleration

$$\textit{change in motion} = \frac{\text{change in velocity}}{\text{elapsed time}} \equiv \textbf{average acceleration}$$

The mathematical form of our acceleration model is:

$$\vec{a}_{avg} \equiv \frac{\Delta \vec{v}}{\Delta t} \quad or \quad \mathbf{a}_{avg} \equiv \frac{\Delta \mathbf{v}}{\Delta t}$$

The SI units that are associated with velocity are:

$$\frac{\text{velocity}}{\text{elapsed time}} = \frac{\frac{length}{time}}{time} = (m/s)/s \quad or \quad m/s/s \quad or \quad \frac{m}{s^2}$$

Ex. A car changes its velocity (accelerates) from 0 *m/s* to 60 *m/s* in 6 *s*. What is the *average acceleration* of the car?

$$a_{avg} = \frac{60\frac{m}{s} - 0\frac{m}{s}}{6s}$$

$$a_{avg} = 10\frac{m}{s^2}$$

Ex. A car changes its velocity from 45 *m/s* to 10 *m/s* in 5 *s*. What is the *average acceleration* of the car?

$$a_{avg} = \frac{10\frac{m}{s} - 45\frac{m}{s}}{5s}$$

$$a_{avg} = -7\frac{m}{s^2}$$

Ex. A car's velocity increases 2 *m/s* every second. What is the *average acceleration* of the car?

$$a_{avg} = 2\frac{m}{s^2}$$

Ex. If the velocity of the car remains constant (*doesn't change*) over a given time interval, what is the *average acceleration* of the car?

$$a_{avg} = 0\frac{m}{s^2}$$

Comment

- To produce a change in **speed** (*a scalar*), you must make a change in magnitude.
- To produce a change in **velocity** or **acceleration** (*a vector*), you must make a change in magnitude, direction or both!

Summary

- **Speed** is a measure of the rate of motion of an object based on how long it takes to cover a specific **distance** between two points.
- **Velocity** is a measure of the rate of motion of an object based on how long it takes to cover the **displacement** between two points.
- The **units** for **speed** and **velocity** are exactly the same.
- **Acceleration** measures how fast you are speeding up or slowing down.

My dear brothers, take note of this: Everyone should be quick to listen, slow to speak and slow to become angry, for man's anger does not bring about the righteous life that God desires.

James 1:19-20