Collisions

In every collision involving a closed system:

1) Conservation of momentum is observed
2) Conservation of ***total*** Energy is observed

There are 2 types of collisions:

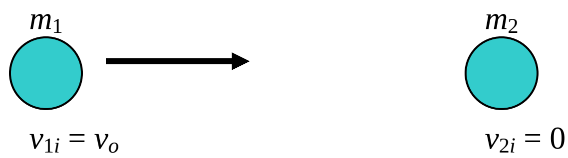
1) Elastic	(*KE is conserved*)	$\Delta KE = 0$
2) Inelastic	(*KE is NOT conserved*)	$\Delta KE \neq 0$

For inelastic collisions, the missing *KE* goes to *heat, deforming, friction …*

Elastic Collision

Ex.

Consider 2 objects of mass m_1 and m_2 colliding elastically. Let m_1 have an initial velocity and m_2 be initially at rest.



What are the final velocities of m_1 and m_2 if:

a) $m_1 = m_2$

b) $m_1 >> m_2$

c) $m_1 << m_2$

From Conservation of momentum

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_o = m_1 v_{1f} + m_2 v_{2f}$$

From Conservation of 'Mechanical' Energy

$$E_i = E_f$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$m_1 v_o^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

Combining the momentum and energy expressions and using a lot of algebra, we find that

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_o$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_o$$

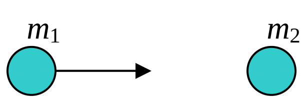
From these expressions, we can determine the motion of the system based on the initial velocity of m_1 and the ratio of the masses.

a) $m_1 = m_2$ (2 *pool balls*)

$v_{1i} = v_o$ $\qquad$ $v_{1f} = 0$

$v_{2i} = 0$ $\qquad$ $v_{2f} = v_o$

Before:



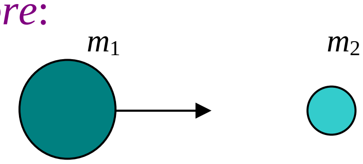
After:

b) $m_1 >> m_2$ (*bowling ball hitting a stationary ping pong ball*)

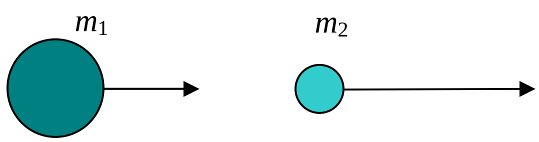
$v_{1i} = v_o$ $v_{1f} \approx v_o$

$v_{2i} = 0$ $v_{2f} \approx 2v_o$

Before:



After:

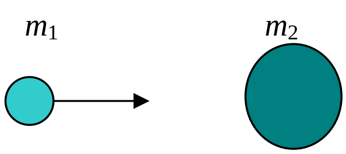


c) $m_1 << m_2$ (*ping pong ball hitting a stationary bowling ball*)

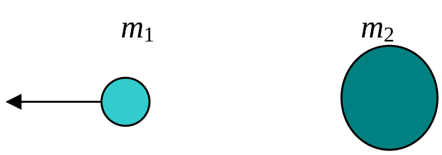
$v_{1i} = v_o$ $v_{1f} \approx -v_o$ (*reverses direction*)

$v_{2i} = 0$ $v_{2f} \approx 0$

Before:



After:



Note: m_2 will actually move if m_1 is not << m_2.

Ex.

Consider 2 objects of mass m_1 and m_2 colliding elastically. Let m_1 have an initial velocity toward m_2 and m_2 have an initial velocity toward m_1.

What are the final velocities of m_1 and m_2 in terms of v_{1i} and v_{2i}?

From Conservation of momentum

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$ (*minus signs for v will be added later*)

From Conservation of 'Mechanical' Energy

$$E_i = E_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

After combining the 2 expressions and much algebra, we get:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Note: If $v_{2i} = 0$, we get back our previous examples expressions for the final velocities:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

What are the final velocities of m_1 and m_2 if $v_o > |-v_o'|$ &:

$a)\ m_1 = m_2$

$b)\ m_1 >> m_2$

$c)\ m_1 << m_2$

$a)\quad m_1 = m_2$ (*2 pool balls*)

$v_{1i} = v_o$ $\qquad v_{1f} = -v_o'$

$v_{2i} = -v_o'$ $\qquad v_{2f} = v_o$ (*reverses direction*)

Before:

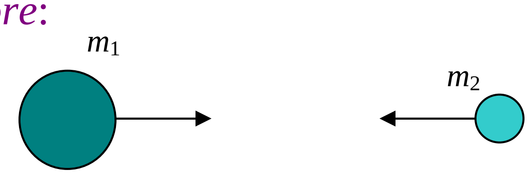
After:

$b)\quad m_1 >> m_2$ (*bowling ball hitting a slow moving ping pong ball*)

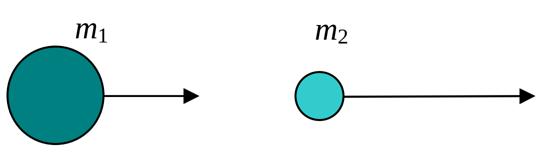
$v_{1i} = v_o$ $\qquad v_{1f} \approx v_o$

$v_{2i} = -v_o'$ $\qquad v_{2f} \approx 2v_o + v'_o$ (*reverses direction*)

Before:



After:

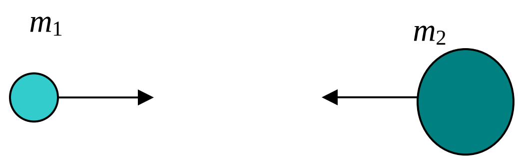


c) $m_1 << m_2$ (*ping pong ball hitting a slow moving bowling ball*)

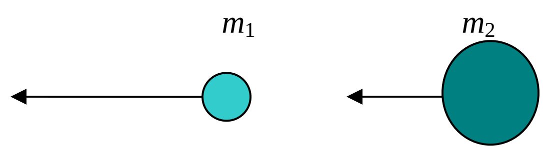
$v_{1i} = v_o$ $v_{1f} \approx -v_o - 2v_o'$ (*reverses direction*)

$v_{2i} = -v_o'$ $v_{2f} \approx -v'_o$

Before:



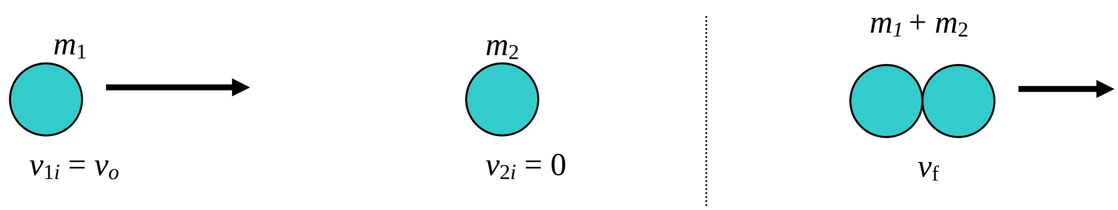
After:



Inelastic Collision

Ex.

Consider 2 objects of mass m_1 and m_2 colliding inelastic-ly. Let m_1 have an initial velocity and m_2 be initially at rest and afterward, they both move stuck together with the same velocity.



Before *After*

From Conservation of momentum

$$p_i = p_f$$

$$m_1 v_{1i} = (m_1 + m_2) v_f$$

From Conservation of 'Mechanical' Energy

$$E_i = E_f$$

$$\frac{1}{2}m_1 v_{1i}^{\ 2} = \frac{1}{2}(m_1 + m_2)v_{1f}^{\ 2}$$

Combining these 2 expressions and after a little algebra:

$$\frac{(m_1 + m_2)}{m_1}v_f^{\ 2} = (m_1 + m_2)v_{1f}^{\ 2}$$

➔ $m_1 + m_2 = m_1$ This statement is False!!!

Ex.

Let $m_1 = m$ & $m_2 = 2m$

From the previous expression, this would yield

$$m + 2m = m$$
$$3m = m$$ **?!?!?**
$$3 = 1$$

This means that KE was **NOT** conserved in the *inelastic* collision. The problem could be fixed using conservation of 'total' energy:

$$\frac{1}{2}m_1 v_{1i}^{\ 2} = \frac{1}{2}(m_1 + m_2)v_{1f}^{\ 2} + E_{loss}$$

Now we would have to come up with some way to represent E_{loss}.