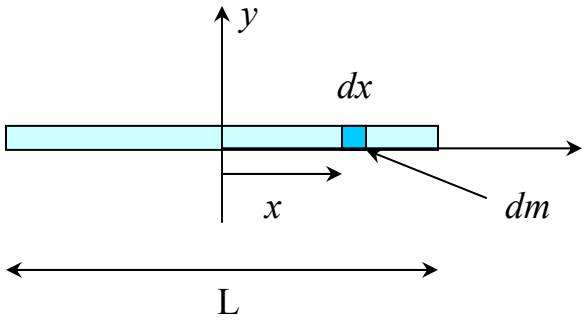
# Moment of Inertia

**Uniform Rigid Rod (*Length* L *and Mass* M) rotating about its midpoint centered on the *y*-axis**




**NOTE:** $l = \mathrm{L}$ & $m = \mathrm{M}$

From the expression for linear mass density:

$$\lambda = \frac{M}{L} \quad \rightarrow \quad \lambda l = m \quad \rightarrow \quad dm = \lambda dl = \lambda dx$$

$$I_y = \int r^2 dm = \int_{\frac{-L}{2}}^{\frac{L}{2}} x^2 \lambda dx$$

$$= \lambda \int_{\frac{-L}{2}}^{\frac{L}{2}} x^2 dx$$

$$= \lambda \left( \frac{1}{3} x^3 \Big|_{\frac{-L}{2}}^{\frac{L}{2}} \right)$$

$$= \frac{\lambda}{3} \left( \frac{L^3}{8} - \frac{-L^3}{8} \right)$$

$$= \frac{\lambda}{3} \left( \frac{L^3}{4} \right)$$

$$= \frac{M}{L} \left( \frac{L^3}{12} \right)$$

$$= \frac{1}{12} ML^2$$