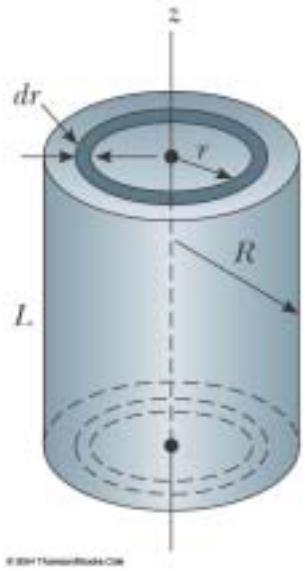
# Moment of Inertia

Uniform solid cylinder rotating about its long axis which is centered on the z-axis




$V_{cyl} = \pi r^2 h = \pi r^2 L$

➔ $dV = 2\pi r L dr$
(*for tall dr ring*)

$$I_z = \int \rho r^2 dV$$

$$= \int_0^R (2\rho\pi L) r^3 dr$$

$$= 2\rho\pi L \int_0^R r^3 dr$$

$$= 2\rho\pi L \left(\frac{1}{4} R^4\right)$$

$$= \frac{1}{2} \rho\pi L R^4$$

* If we want to reduce our answer to an expression that only includes M & R.

Using $\rho = \frac{M}{V}$ for the total cylinder and the value for V for a cylinder

$$\rho = \frac{M}{\pi r^2 L}$$

➔ $$I_z = \frac{1}{2}\left(\frac{M}{\pi R^2 L}\right)\pi L R^4$$

$$I_z = \frac{1}{2} M R^2$$