Analyzing Motion Using Newton's Laws

Consider 2 Skaters in the middle of a frictionless frozen lake:

a 275 *lb* (125 *kg*) *football player* (m_{fp}) & *a* 100 *lb* (45 *kg*) *figure skater* (m_{fs}))

* If the two skaters are facing each other with hands together initially at rest, what is the magnitude of the force acting on each? **0** **(Newton's 1st Law)**

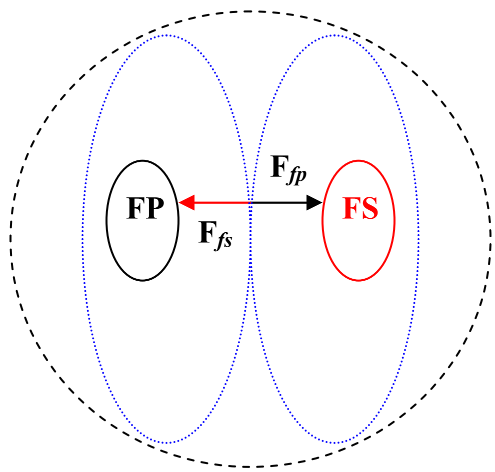
* Do the skaters move apart if they push off? ***Yes*** **(Newton's 2nd Law)**

* How can this be if the sum of their forces is zero (**Newton's 2nd Law**)?

➔ *The net force for our total interacting SYSTEM is zero, but the individual forces acting on different objects within the system is non-zero.*

Let's model this situation or system using a simple diagram.

The 2 interacting objects in this system are the 2 skaters



Where is the 3rd law being applied? (*black dashed line*)
Where is the 2nd law being applied? (*blue dotted lines*)

* Which skater exerts the larger force?

(***The magnitude of the forces is the same, but in opposite directions!*** **[*3rd law*]**)

$$\mathbf{F}_{\text{Football player}} = -\,\mathbf{F}_{\text{figure skater}}$$

Can we use all of Newton's Laws to predict the motion of the 2 skaters after they push off? YES!

From Newton's 3rd Law we know that:

$$\mathbf{F}_{fp} = -\mathbf{F}_{fs}$$

Using Newton's 2nd Law ($F = ma$) *and applying it to each skater, we find*:

$$m_{fp}\boldsymbol{a}_{fp} = -m_{fs}\boldsymbol{a}_{fs}$$

Analysis:

1) This statement implies that the change in motion (*or the acceleration*) of the skaters is in opposite directions.

2) The magnitudes of both sides are equal.

Thus:

if m_{fp} ~ big & m_{fs} ~ small

→ $|\boldsymbol{a}_{fp}|$ ~ small & $|-\boldsymbol{a}_{fs}|$ ~ big **so that their product is the same**

Using one of our kinematic equations of motion ($v_f = v_i + at$), we can make an educated guess as to magnitude of their final velocities [noting that $v_i = 0$]:

$|\boldsymbol{v}_{fp}|$ ~ small & $|-\boldsymbol{v}_{fs}|$ ~ big

Conclusion: A faster acceleration will lead to a faster final velocity. Thus, the figure skater will have a faster final speed at any given time.

* What would the motion of both skaters be if their masses were equal?

The magnitudes of their acceleration and velocities would be the same, but still in opposite directions (*i.e. they would get off at the same time*).

Ex.

The 2 skaters, initially at rest, push off by exerting a force equal to 225 N on each other. If the $m_{fp} = 125\ kg$ and the $m_{fs} = 45\ kg$, what are their speeds after 5 s?

FP: $F = m_{fp}a_{fp}$

$$a_{fp} = \frac{F}{m_{fp}} = \frac{225N}{125kg} \quad \rightarrow \quad a_{fp} = 1.8\frac{m}{s^2}$$

FS: $F = m_{fs}a_{fs}$

$$a_{fs} = \frac{F}{m_{fs}} = \frac{225N}{45kg} \quad \rightarrow \quad a_{fs} = 5\frac{m}{s^2}$$

Speeds:

FP: $v_f = v_i + at$ ➔ $v_f = \left(1.8\frac{m}{s^2}\right)(5s) = 9\frac{m}{s}$

FS: $v_f = v_i + at$ ➔ $v_f = \left(5\frac{m}{s^2}\right)(5s) = 25\frac{m}{s}$

* Which skater will slow down the quickest after they have pushed off?

Neither, because of Newton's 1st Law (*since we assumed no friction*)