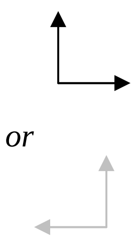
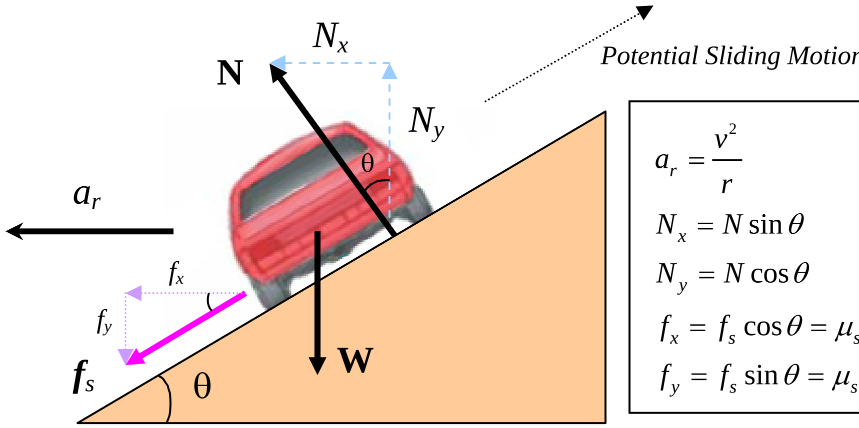
Cars Traveling Around a Banked Curve (*w/ friction*)

Ex.

Find the maximum speed a car of mass *m* traveling along a banked curve (*whose path is the shape of a circle of radius r*) can have in order to make the curve without sliding **up** the incline.





$$a_r = \frac{v^2}{r}$$
$$N_x = N\sin\theta$$
$$N_y = N\cos\theta$$
$$f_x = f_s\cos\theta = \mu_s N\cos\theta$$
$$f_y = f_s\sin\theta = \mu_s N\sin\theta$$

Determine the motion in each direction using Newton's 2nd law and the force diagram.

$$\sum F_x = -ma_r \qquad\qquad \sum F_y = 0$$

$$-N_x - f_x = -ma_r \qquad\qquad N_y - W - f_y = 0$$

Substitute & solve for the Normal Force from the *y* component:

$$-N\sin\theta - \mu_s N\cos\theta = -m\left(\frac{v^2}{r}\right) \qquad\qquad N\cos\theta - mg - \mu_s N\sin\theta = 0$$

$$N\left(\sin\theta + \mu_s\cos\theta\right) = m\left(\frac{v^2}{r}\right) \qquad\qquad N\left(\cos\theta - \mu_s\sin\theta\right) = mg$$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

Substitute the expression for the Normal force into the *x* component equation and solve for *v*:

$$\left(\frac{mg}{\cos\theta - \mu_s\sin\theta}\right)\left(\sin\theta + \mu_s\cos\theta\right) = m\left(\frac{v^2}{r}\right)$$

$$\frac{v^2}{r} = \frac{g\left(\sin\theta + \mu_s\cos\theta\right)}{\cos\theta - \mu_s\sin\theta}$$

$$v_{max} = \sqrt{\frac{gr\left(\sin\theta + \mu_s\cos\theta\right)}{\cos\theta - \mu_s\sin\theta}}$$

Including friction, this is the restriction on the speed of the car to go around a banked curve without sliding up the incline.

If $v_{car} > v_{max}$, the car will slide up the incline

In terms of the angle:

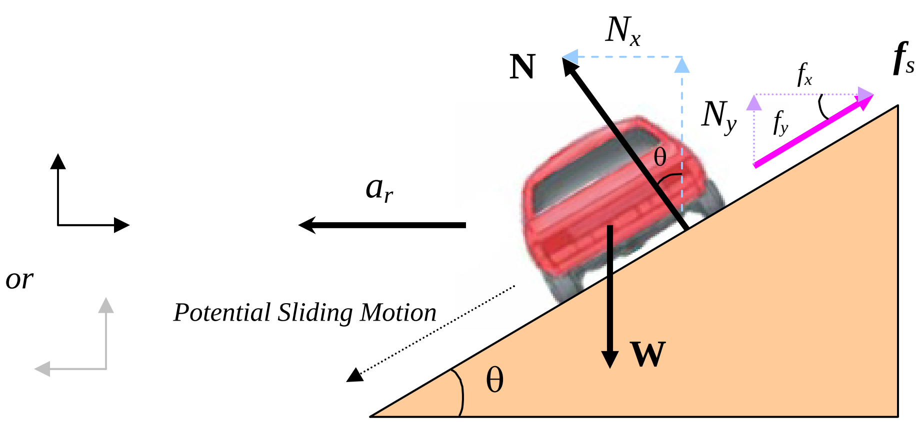
$$\tan\theta = \frac{\left(\frac{v^2}{gr}\right) - \mu_s}{1 + \left(\frac{\mu_s v^2}{gr}\right)}$$

*** There is also a *minimum* speed the car must have when you include friction or it will start to slide <u>down</u> the incline.**

For this scenario, the potential motion is down the incline. This means the static friction force points up the incline.

Ex.

Find the minimum speed a car traveling along a banked curve (*whose path is the shape of a circle*) can have in order to make the curve without sliding **<u>down</u>** the incline.



$$a_r = \frac{v^2}{r}$$

$$N_x = N\sin\theta$$

$$N_y = N\cos\theta$$

$$f_x = f_s\cos\theta = \mu_s N\cos\theta$$

$$f_y = f_s\sin\theta = \mu_s N\sin\theta$$

Determine the motion in each direction using Newton's 2nd law and the force diagram.

$$\sum F_x = -ma_r \qquad \sum F_y = 0$$

$$-N_x + f_x = -ma_r \qquad N_y - W + f_y = 0$$

Substitute & solve for the Normal Force from the *y* component:

$$-N\sin\theta + \mu_s N\cos\theta = -m\left(\frac{v^2}{r}\right) \qquad N\cos\theta - mg + \mu_s N\sin\theta = 0$$

$$N\left(\sin\theta - \mu_s\cos\theta\right) = m\left(\frac{v^2}{r}\right) \qquad N\left(\cos\theta + \mu_s\sin\theta\right) = mg$$

$$N = \frac{mg}{\cos\theta + \mu_s\sin\theta}$$

Substitute the expression for the Normal force into the *x* component equation and solve for *v*:

$$\left(\frac{mg}{\cos\theta+\mu_s\sin\theta}\right)(\sin\theta-\mu_s\cos\theta)=m\left(\frac{v^2}{r}\right)$$

$$\frac{v^2}{r}=\frac{g(\sin\theta-\mu_s\cos\theta)}{\cos\theta+\mu_s\sin\theta}$$

$$v_{\min}=\sqrt{\frac{gr(\sin\theta-\mu_s\cos\theta)}{\cos\theta+\mu_s\sin\theta}}$$

Including friction, this is the restriction on the speed of the car to go around a banked curve without sliding down the incline.

If $v_{car} < v_{min}$, the car will slide down the incline

In terms of the angle:

$$\tan\theta=\frac{\left(\frac{v^2}{gr}\right)+\mu_s}{1-\left(\frac{\mu_s v^2}{gr}\right)}$$