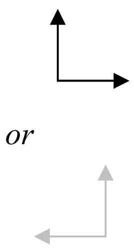
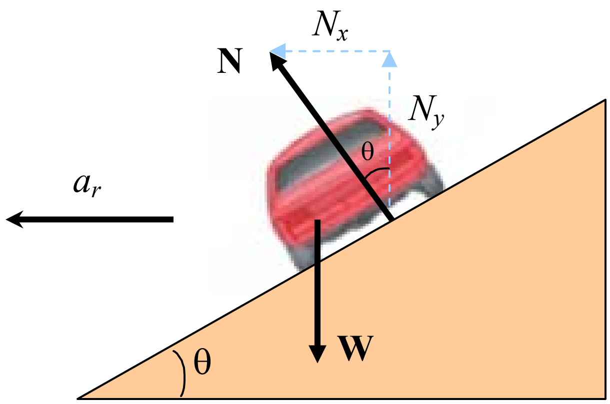
Cars Traveling Around a Banked Curve (*no friction*)

NOTE: In these types of problems, always choose a coordinate system such that x is *parallel* or *anti-parallel* to a_r.

Ex.

Find the exact speed a car of mass m traveling along a banked curve (*whose path is the shape of a circle of radius r*) must have in order to make the curve without sliding **up** or **down** the incline.





$$a_r = \frac{v^2}{r}$$
$$N_x = N\sin\theta$$
$$N_y = N\cos\theta$$

Determine the motion in each direction using Newton's 2nd law and the force diagram.

$$\sum F_x = -ma_r \qquad\qquad \sum F_y = 0$$

$$-N_x = -ma_r \qquad\qquad N_y - W = 0$$

Substituting & solving for the Normal Force:

$$-N\sin\theta = -m\left(\frac{v^2}{r}\right) \qquad\qquad N\cos\theta - mg = 0$$

$$N = \frac{mv^2}{r\sin\theta} \qquad\qquad N = \frac{mg}{\cos\theta}$$

Equating the two expressions for the Normal force:

$$\frac{mv^2}{r\sin\theta} = \frac{mg}{\cos\theta}$$

$$v^2 = \frac{rg\sin\theta}{\cos\theta}$$

$$v = \sqrt{rg\tan\theta}$$

W/o friction, this is the restriction on the speed of the car to go around a banked curve without sliding up or down the incline.

If $v_{car} > v$, the car will slide up the incline

If $v_{car} < v$, the car will slide down the incline

In terms of the angle:

$$\tan\theta = \frac{v^2}{rg}$$