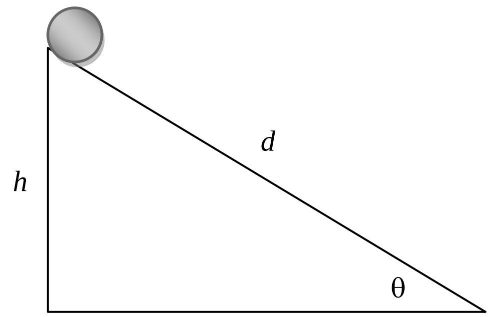
# Conservation of Energy Investigation – Derivations

## Solid Sphere $\left(I = \frac{2}{5}mR^2\right)$

**Velocity:**




$$\sin\theta = \frac{h}{d}$$

$$v = R\omega$$

$$E_i = E_f$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}m\left(R^2\omega^2\right)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}m\left(v^2\right)$$

$$mgh = \frac{7}{10}mv^2$$

$$gh = \frac{7}{10}v^2$$

$$v = \sqrt{\left(\frac{10}{7}\right)gh}$$

**Acceleration:**

Using $v^2 = v_o{}^2 + 2ad$

$$a = \frac{v^2}{2d}$$

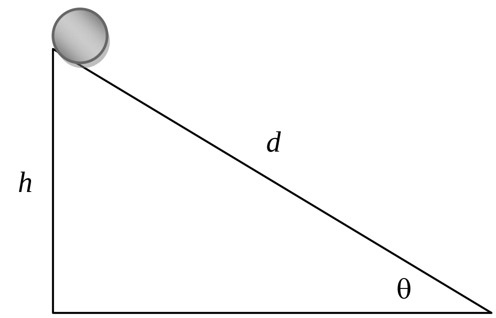
$$a = \frac{\left(\sqrt{\left(\frac{10}{7}\right)gh}\right)^2}{2d}$$

$$a = \left(\frac{5}{7}\right)g\left(\frac{h}{d}\right)$$

$$a = \left(\frac{5}{7}\right)g\sin\theta$$

## Hollow Sphere $\left(I = \frac{2}{3}mR^2\right)$

**Velocity:**




$$\sin\theta = \frac{h}{d}$$

$$v = R\omega$$

$$E_i = E_f$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{3}m\left(R^2\omega^2\right)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{3}m\left(v^2\right)$$

$$mgh = \frac{5}{6}mv^2$$

$$gh = \frac{5}{6}v^2$$

$$v = \sqrt{\left(\frac{6}{5}\right)gh}$$

**Acceleration:**

Using $v^2 = v_o{}^2 + 2ad$

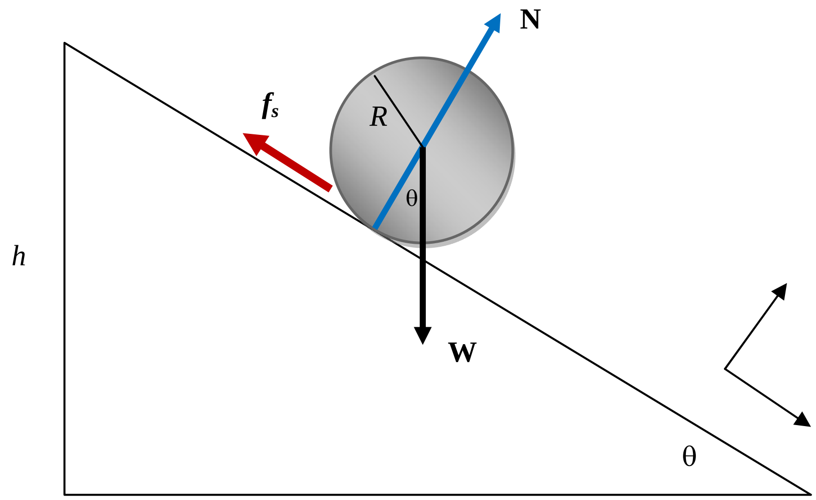
$$a = \frac{v^2}{2d}$$

$$a = \frac{\left(\sqrt{\left(\frac{6}{5}\right)gh}\right)^2}{2d}$$

$$a = \left(\frac{3}{5}\right)g\left(\frac{h}{d}\right)$$

$$a = \left(\frac{3}{5}\right)g\sin\theta$$

**The accelerations can also be found using the sum of the forces.**




**x:**

$$mg\sin\theta - f_s = ma$$

**y:**

$$N - mg\cos\theta = 0$$

Applying the rotational equivalent of Newton's Second Law and the moment of inertia for a *solid sphere*:

$$\tau = I\alpha$$

$$f_s R = \left(\frac{2}{5} mR^2\right)\alpha \qquad \textit{using} \quad \tau = Fr\sin\theta = f_s R(1) = f_s R$$

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$$f_s = \frac{2}{5} mR\alpha$$

$$f_s = \frac{2}{5} m\left(R\alpha\right)$$

$$f_s = \frac{2}{5} ma$$

Combining this expression with the force expression in the x direction yields:

$$mg\sin\theta - \frac{2}{5} ma = ma$$

$$g\sin\theta = \frac{7}{5} a$$

$$a = \frac{7}{5} g\sin\theta$$

*The same process can be used to find the acceleration for a hollow sphere or any other object.*