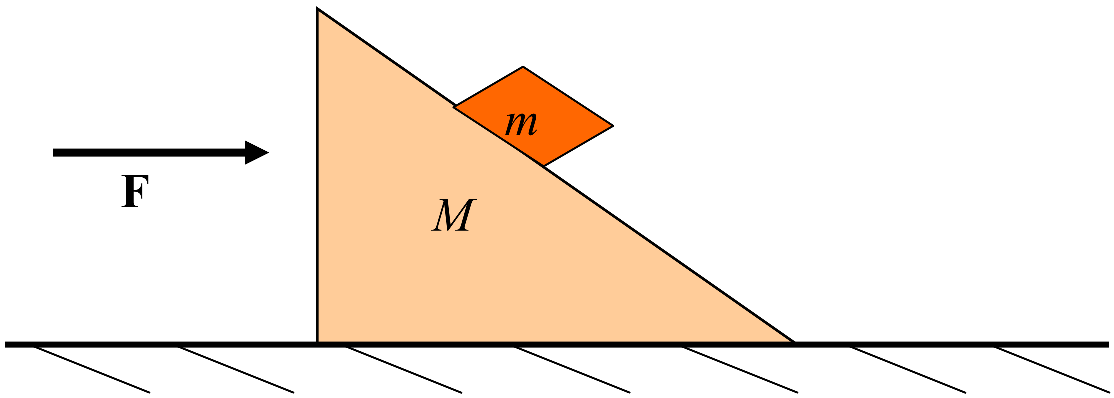
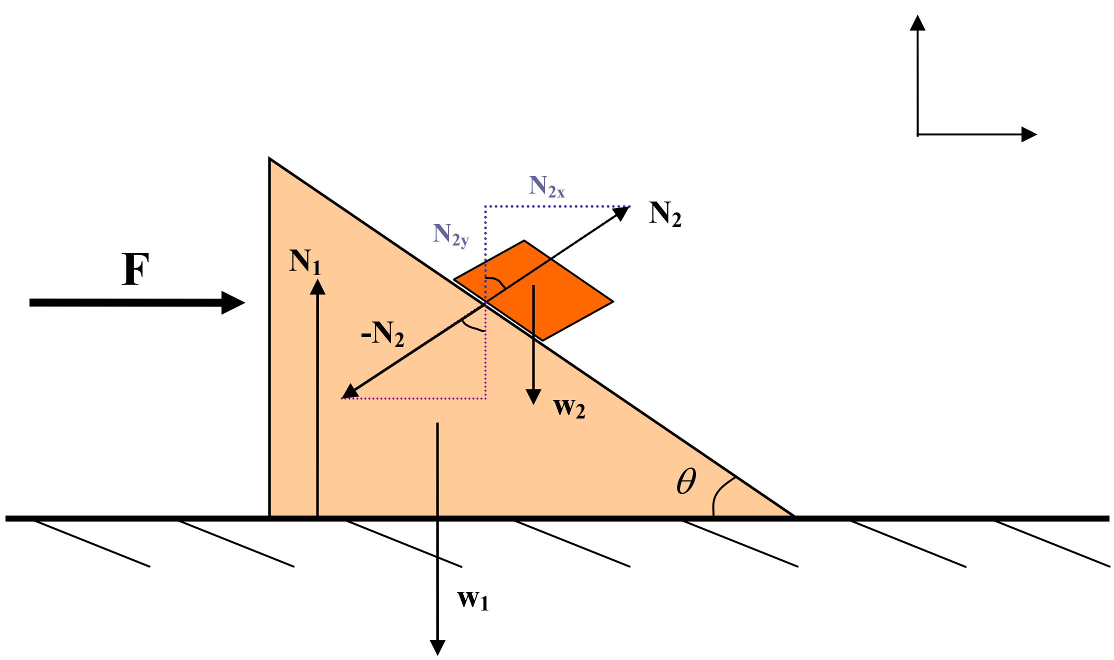
Complex Force Problem (*No Friction*)



A small block of mass *m* is sits on an incline of mass *M* and is free to move. A force pushes on the large incline accelerating it forward. What is the magnitude of the force required to keep the small block from sliding up or down the incline?

NOTE: Since *m* does not move up or down the incline while *M* is in motion, this implies that *m* and *M* both have the same horizontal velocity & acceleration.

Force Diagram



From the Force Diagram, we can see that:

$w_{2x} = 0$ $\qquad$ $N_{2x} = N_2\sin\theta$

$w_{2y} = -mg$ $\qquad$ $N_{2y} = N_2\cos\theta$

*** Forces acting on the *m* system:**

$$\sum F_{2x} = ma_{2x}$$
$$N_{2x} + w_{2x} = ma_{2x}$$
$$N_2 \sin\theta = ma_{2x}$$

$$N_2 = \frac{ma_{2x}}{\sin\theta}$$

$$\sum F_{2y} = 0 \quad (\textit{no motion in the y direction})$$
$$N_{2y} + w_{2y} = 0$$
$$N_2 \cos\theta - mg = 0$$

$$N_2 = \frac{mg}{\cos\theta}$$

Setting equal:

$$\frac{ma_{2x}}{\sin\theta} = \frac{mg}{\cos\theta}$$

$$a_{2x} = g\frac{\sin\theta}{\cos\theta} \quad \Rightarrow \quad a_{2x} = g\tan\theta$$

*** Forces acting on the *M* system:**

$$\sum F_{1x} = Ma_{1x}$$
$$F + N_{1x} + w_{1x} - N_{2x} = Ma_{1x}$$

$$\sum F_{1y} = 0 \quad (\textit{no motion in the y direction})$$
$$N_{1y} - w_{1y} - N_{2y} = 0$$

From the Force Diagram, we can see that:

$w_{1x} = 0$ $\qquad$ $N_{1x} = 0$

&

From Newton's 3rd Law, the normal force acting on *m* due to *M* (**N**$_2$) is equal and in the opposite direction as the normal force acting on *M* due to *m* (-**N**$_2$).

$N_{2x} = - N_{2x}$ $\qquad$ & $\qquad$ $N_{2y} = - N_{2y}$

In system *m* $\quad$ In system *M* $\qquad$ In system *m* $\quad$ In system *M*

Substituting:

$$\mathrm{F} - \mathrm{N}_2 \sin\theta = Ma_{1x} \qquad \mathrm{N}_{1y} - Mg - \mathrm{N}_2 \cos\theta = 0$$

$$\mathrm{F} = Ma_{1x} + \mathrm{N}_2 \sin\theta \qquad \mathrm{N}_{1y} = Mg + \mathrm{N}_2 \cos\theta$$

Since the 2 blocks have the same horizontal acceleration:

$$|a_{1x}| = |a_{2x}| = a$$

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Combining this constraint with our expression for F, $a_{2x} = g\tan\theta$ and $\mathrm{N}_2 = \dfrac{mg}{\cos\theta}$, we get:

$$\mathrm{F} = Mg\tan\theta + \left(\frac{mg}{\cos\theta}\right)\sin\theta$$

$$\mathrm{F} = Mg\tan\theta + mg\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\mathrm{F} = Mg\tan\theta + mg\tan\theta$$

$$\underline{\mathrm{F} = (M+m)g\tan\theta}$$