Complex Force Problem *(With Friction)*

A small block of mass *m* is sits on an incline of mass *M* and is free to move. A force pushes on the large incline accelerating it forward. What is the magnitude of the minimum force required to start the block moving up the incline?

**Force Diagram**
From the Force Diagram, we can see that:

\[ w_{2x} = 0 \quad \text{N} \]
\[ N_{2x} = N_2 \sin \theta \]
\[ w_{2y} = -mg \quad \text{N} \]
\[ N_{2y} = N_2 \cos \theta \]

* Forces acting on the \( m \) system:

If the block is stationary on the incline and but is accelerating to the right, the static friction force \( f_s \) points up the incline to keep it from sliding down the surface. Under these conditions:

\[ f_s < \mu_s N_2 \quad \text{&} \quad |a_{1x}| = |a_{2x}| = a : \]

\[ \sum F_{2x} = ma_{2x} \quad \text{&} \quad \sum F_{2y} = 0 \quad (\text{no motion in the y direction}) \]
\[ N_{2x} - w_{2x} - f_s = ma_{2x} \quad \text{&} \quad N_{2y} - w_{2y} + f_s = 0 \]
\[ N_2 \sin \theta - f_s \cos \theta = ma \quad \text{&} \quad N_2 \cos \theta - mg - f_s \sin \theta = 0 \]

Find the maximum value for \( a \) for which \( f_s = \mu_s N_2 \) (on the verge of moving):

\[ N_2 \cos \theta - mg - \mu_s N_2 \sin \theta = 0 \]
\[ N_2 \left( \cos \theta - \mu_s \sin \theta \right) = mg \]
\[ N_2 = \frac{mg}{\left( \cos \theta - \mu_s \sin \theta \right)} \]

\[ a_{\max} = \frac{N_2 \sin \theta - \mu_s N_2 \cos \theta}{m} \]
\[ a_{\max} = \frac{N_2 \left( \sin \theta - \mu_s \cos \theta \right)}{m} \]
\[ a_{\max} = \left( \frac{mg}{\left( \cos \theta - \mu_s \sin \theta \right)} \right) \left( \frac{\sin \theta - \mu_s \cos \theta}{m} \right) \]
\[ a_{\max} = \frac{g \left( \sin \theta - \mu_s \cos \theta \right)}{\left( \cos \theta - \mu_s \sin \theta \right)} \]
**Forces acting on the M system:**

Since F is only in the x direction, we only need to look at the forces acting on M in the x direction in order to find $F_{min}$. In addition, since the intended motion of $m$ is up the incline, the static friction force ($f_s$) points down the incline to keep it from sliding up the surface.

From the Force Diagram, we can see that:

\[ w_{1x} = 0 \quad N_{1x} = 0 \]

&

From Newton’s 3rd Law, the normal force acting on $m$ due to $M$ ($N_2$) is equal and in the opposite direction as the normal force acting on $M$ due to $m$ (-$N_2$).

\[ N_{2x} = -N_{2x} \quad \text{In system } m \quad \text{In system } M \]

\[ \sum F_{lx} = Ma_{lx} \]

\[ F_{min} + N_{1x} + w_{1x} - N_{2x} + f_s \cos \theta = Ma_{lx} \]

\[ F_{min} - N_2 \sin \theta + f_s \cos \theta = Ma \]

In order for the small block to start moving up the incline, $a > a_{max}$.

\[ F_{min} - N_2 \sin \theta + \mu_s N_2 \cos \theta > Ma_{max} \]

\[ F_{min} > Ma_{max} + N_2 (\sin \theta - \mu_s \cos \theta) \]

But

\[ N_2 (\sin \theta - \mu_s \cos \theta) = ma_{max} \quad \text{from } N_2 \sin \theta - f_s \cos \theta = ma \quad \text{(in the m system)} \]

\[ F_{min} > Ma_{max} + ma_{max} \]

\[ F_{min} > (m + M)a_{max} \]

\[ F_{min} > \frac{g(m + M)(\sin \theta - \mu_s \cos \theta)}{(\cos \theta - \mu_s \sin \theta)} \]