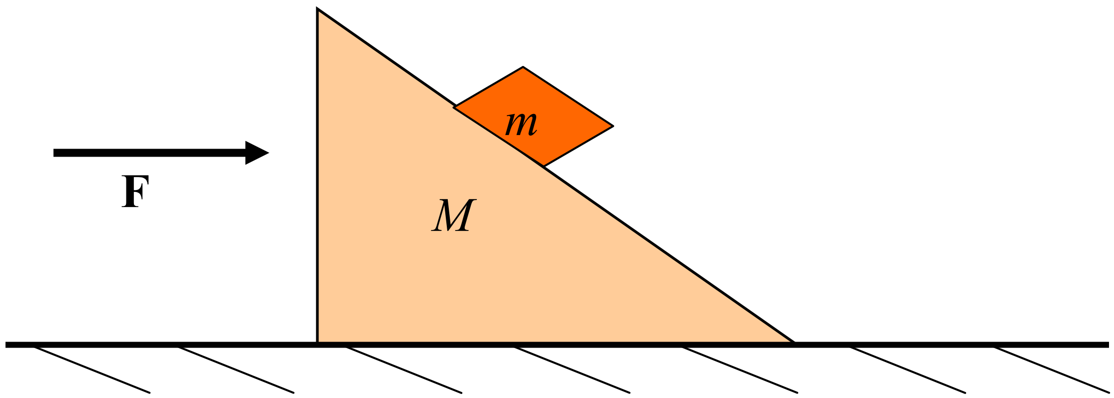
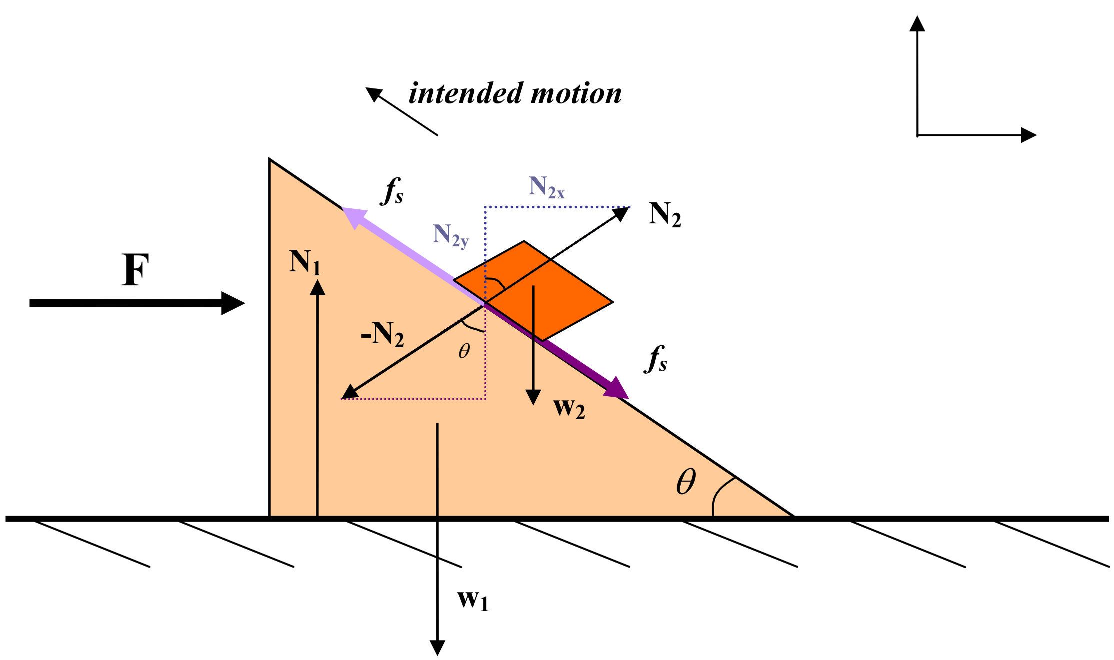
Complex Force Problem (*With Friction*)



A small block of mass *m* is sits on an incline of mass *M* and is free to move. A force pushes on the large incline accelerating it forward. What is the magnitude of the minimum force required to start the block moving up the incline?

Force Diagram



From the Force Diagram, we can see that:

$w_{2x} = 0$ $\quad\quad$ $N_{2x} = N_2\sin\theta$

$w_{2y} = -mg$ $\quad\quad$ $N_{2y} = N_2\cos\theta$

*** <u>Forces acting on the *m* system</u>:**

If the block is stationary on the incline and but is accelerating to the right, the static friction force (f_s) points up the incline to keep it from sliding down the surface. Under these conditions:

$f_s < \mu_s N_2$ $\quad$ & $\quad$ $|a_{1x}| = |a_{2x}| = a$:

$$\sum F_{2x} = ma_{2x}$$

$$N_{2x} - w_{2x} - f_s = ma_{2x}$$

$$N_2\sin\theta - f_s\cos\theta = ma$$

$$\sum F_{2y} = 0 \quad (\textit{no motion in the y direction})$$

$$N_{2y} - w_{2y} + f_s = 0$$

$$N_2\cos\theta - mg - f_s\sin\theta = 0$$

Find the maximum value for a for which $f_s = \mu_s N_2$ (*on the verge of moving*):

$$N_2\cos\theta - mg - \mu_s N_2\sin\theta = 0$$

$$N_2(\cos\theta - \mu_s\sin\theta) = mg$$

$$N_2 = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

➔

$$a_{\max} = \frac{N_2\sin\theta - \mu_s N_2\cos\theta}{m}$$

$$a_{\max} = \frac{N_2(\sin\theta - \mu_s\cos\theta)}{m}$$

$$a_{\max} = \left(\frac{mg}{(\cos\theta - \mu_s\sin\theta)}\right)\frac{(\sin\theta - \mu_s\cos\theta)}{m}$$

$$a_{\max} = \frac{g(\sin\theta - \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$

* **Forces acting on the *M* system:**

Since F is only in the x direction, we only need to look at the forces acting on M in the x direction in order to find F_{min}. In addition, since the intended motion of m is up the incline, the static friction force ($\boldsymbol{f_s}$) points down the incline to keep it from sliding up the surface.

From the Force Diagram, we can see that:

$w_{1x} = 0$ $\quad\quad$ $N_{1x} = 0$

&

From Newton's 3rd Law, the normal force acting on m due to M ($\mathbf{N}_2$) is equal and in the opposite direction as the normal force acting on M due to m ($-\mathbf{N}_2$).

$$\underbrace{N_{2x}}_{\text{In system } m} = \underbrace{-\,N_{2x}}_{\text{In system } M}$$

➔

$$\sum F_{1x} = Ma_{1x}$$

$$F_{\min} + N_{1x} + w_{1x} - N_{2x} + f_s\cos\theta = Ma_{1x}$$

$$F_{\min} - N_2\sin\theta + f_s\cos\theta = Ma$$

In order for the small block to start moving up the incline, $a > a_{max}$.

$$F_{\min} - N_2\sin\theta + \mu_s N_2\cos\theta > Ma_{\max}$$

$$F_{\min} > Ma_{\max} + N_2(\sin\theta - \mu_s\cos\theta)$$

But

$$N_2(\sin\theta - \mu_s\cos\theta) = ma_{\max} \qquad \textit{from } N_2\sin\theta - f_s\cos\theta = ma \ (\textit{in the m system})$$

➔

$$F_{\min} > Ma_{\max} + ma_{\max}$$

$$F_{\min} > (m+M)a_{\max}$$

$$F_{\min} > \frac{g(m+M)(\sin\theta - \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$