Finding the Force on an Elevator Cable when:

I. The elevator is at rest
II. The elevator accelerates upward from rest
III. The elevator accelerates downward from rest
IV. The elevator is moving at a constant velocity
V. The elevator is moving upward and comes to rest
VI. The elevator is moving downward and comes to rest

There are 2 approaches for solving this problem using forces:

Method I
Determine the sign of the acceleration and incorporate it into the force and kinematic equations

Method II
Assume the acceleration is positive in \( F = ma \) for all cases and then let its sign be determined from the kinematic equations based on the information given in the problem

Method I

Whenever a force is directed along a linear object such as a cable, string...it is typically referred to as the Tension (force), \( T \).

I. The elevator is at rest

The forces acting on the elevator can be represented by:

\[
\sum F = 0
\]

\[
T - mg = 0 \quad \Rightarrow \quad T = mg
\]

\[
= (800 \text{ kg})(9.8 \text{ m/s}^2) = 7840 \text{ N}
\]
II. The elevator accelerates upward from rest

\[ \Sigma F = ma \]
\[ T - mg = ma \]  \text{(note: } ma \text{ is positive since } a \text{ is in the + direction)}
\[ \Rightarrow \quad T = mg + a \]

* Need value of \( a \):
\[
\begin{array}{c|c|c}
\text{Given} & \text{Needed} & \text{Result} \\
\hline
v_0 & v & \Delta x & a & v^2 = v_0^2 + 2a\Delta x \\
\end{array}
\]

\[ a = \frac{v^2}{2\Delta x} = \frac{(10 \text{ m/s})^2}{2(25 \text{ m})} \]
\[ a = 2 \text{ m/s}^2 \]

Therefore,
\[ T = mg + a \]
\[ = (800 \text{ kg})(9.8 \text{ m/s}^2 + 2 \text{ m/s}^2) = 9440 \text{ N} \]

III. The elevator accelerates downward from rest

\[ \Sigma F = ma \]
\[ T - mg = -ma \]  \text{(note: } ma \text{ is negative since } a \text{ is in the – direction)}
\[ \Rightarrow \quad T = mg - a \]
* Need value of $a$:

<table>
<thead>
<tr>
<th>Given</th>
<th>Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>$a$</td>
</tr>
<tr>
<td>$v$</td>
<td>$\Delta x$</td>
</tr>
</tbody>
</table>

$$v^2 = v_0^2 - 2a\Delta x \quad (noting\ that\ a < 0)$$

**Solving for $a$:**

$$a = \frac{-v^2}{2\Delta x} = \frac{-(10\ m/s)^2}{2(-25\ m)} \quad (note:\ \Delta x < 0\ since\ motion\ is\ in\ the\ –\ direction)$$

$$a = 2\ m/s^2$$

Therefore, $T = mg - a$

$$= (800\ kg)(9.8\ m/s^2 - 2\ m/s^2) = 6240\ N$$

**IV. The elevator is moving at a constant velocity**

$$\sum F = 0$$

$T - mg = 0 \quad \Rightarrow \quad T = mg$

$$= (800\ kg)(9.8\ m/s^2) = 7840\ N \quad (same\ as\ if\ the\ elevator\ was\ at\ rest!)$$

**Would the result be different if the direction of motion were reversed?**

No!

**V. The elevator is moving upward and comes to rest**

$$\sum F = 0$$

$T - mg - a = 0 \quad \Rightarrow \quad T = mg - a$

$$= (800\ kg)(9.8\ m/s^2) = 7840\ N \quad (same\ as\ if\ the\ elevator\ was\ at\ rest!)$$

$v_0 = 10\ m/s$

$v = 0\ m/s\ \ after\ moving\ 25\ m$
\[ \sum F = ma \]

\[ T - mg = -ma \quad (\text{note: } ma \text{ is negative since } a \text{ is in the } - \text{ direction}) \]

\[ \Rightarrow \quad T = m(g - a) \]

* Need value of \( a \):

Use \( v^2 = v_0^2 - 2a\Delta x \quad (\text{noting that } a < 0) \)

**Solving for \( a \):**

\[ a = \frac{v_0^2}{2\Delta x} = \frac{(10 \text{ m/s})^2}{2(25 \text{ m})} \]

\[ a = 2 \text{ m/s}^2 \]

Therefore, \( T = m(g - a) \)

\[ = (800 \text{ kg})(9.8 \text{ m/s}^2 - 2 \text{ m/s}^2) = 6240 \text{ N} \]

**VI. The elevator is moving downward and comes to rest**

\[ \sum F = ma \]

\[ T - mg = ma \quad (\text{note: } ma \text{ is positive since } a \text{ is in the } + \text{ direction}) \]

\[ \Rightarrow \quad T = m(g + a) \]

* Need value of \( a \):

Use \( v^2 = v_0^2 + 2a\Delta x \quad (\text{noting that } a > 0) \)

**Solving for \( a \):**

\[ a = \frac{-v_0^2}{2\Delta x} = \frac{-(10 \text{ m/s})^2}{2(-25 \text{ m})} \quad (\text{note: } \Delta x < 0 \text{ since motion is in the } - \text{ direction}) \]

\[ a = 2 \text{ m/s}^2 \]

Therefore, \( T = m(g + a) \)

\[ = (800 \text{ kg})(9.8 \text{ m/s}^2 + 2 \text{ m/s}^2) = 9440 \text{ N} \]
Method II

For all cases, let the forces representing the system be given by:

\[ \sum F = ma \]

\[ T - mg = ma \quad (\text{note: } ma \text{ is positive since } a \text{ is in the + direction}) \]

\[ \Rightarrow \quad T = m(g + a) \]

* Need value of \( a \):

I. The elevator is at rest

\[ a = 0 \text{ m/s}^2 \]

\[ \Rightarrow \quad T = mg \]

\[ = (800 \text{ kg})(9.8 \text{ m/s}^2) = 7840 \text{ N} \]

II. The elevator accelerates upward from rest

\[
\begin{array}{c|c|c}
\text{Given} & \text{Needed} & \text{Equation} \\
\hline
v_0 & v & \Delta x \\
\hline
& a & v^2 = v_0^2 + 2a\Delta x \\
\end{array}
\]

Solving for \( a \):

\[ a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(10)^2}{2(25)} \]

\[ a = 2 \text{ m/s}^2 \]

Therefore, \( T = m(g + a) \)

\[ = (800 \text{ kg})(9.8 \text{ m/s}^2 + 2 \text{ m/s}^2) = 9440 \text{ N} \]
III. The elevator accelerates downward from rest

Using \( v^2 = v_o^2 + 2a\Delta x \)

\[
a = \frac{v^2}{2\Delta x} = \frac{(10 \text{ m/s})^2}{2(-25 \text{ m})}  \quad \text{(note: } \Delta x < 0 \text{ since motion is in the – direction)}
\]

\[a = -2 \text{ m/s}^2\]

Therefore, \( T = m(g + a) \)

\[
= (800 \text{ kg})(9.8 \text{ m/s}^2 - 2 \text{ m/s}^2) = 6240 \text{ N}
\]

IV. The elevator is moving at a constant velocity

\( a = 0 \text{ m/s}^2 \) \hspace{1cm} (regardless of direction)

\( \Rightarrow \ T = mg \)

\[
= (800 \text{ kg})(9.8 \text{ m/s}^2) = 7840 \text{ N}
\]

V. The elevator is moving upward and comes to rest

Using \( v^2 = v_o^2 + 2a\Delta x \)

\[
a = \frac{-v_o^2}{2\Delta x} = \frac{-(10 \text{ m/s})^2}{2(25 \text{ m})}
\]

\[a = -2 \text{ m/s}^2\]

Therefore, \( T = m(g + a) \)

\[
= (800 \text{ kg})(9.8 \text{ m/s}^2 - 2 \text{ m/s}^2) = 6240 \text{ N}
\]

VI. The elevator is moving downward and comes to rest

Using \( v^2 = v_o^2 + 2a\Delta x \)

\[
a = \frac{-v_o^2}{2\Delta x} = \frac{-(10 \text{ m/s})^2}{2(-25 \text{ m})} \quad \text{(note: } \Delta x < 0 \text{ since motion is in the – direction)}
\]

\[a = 2 \text{ m/s}^2\]

Therefore, \( T = m(g + a) \)

\[
= (800 \text{ kg})(9.8 \text{ m/s}^2 + 2 \text{ m/s}^2) = 9440 \text{ N}
\]