Modeling Uniform Motion

We believe that mere movement is life, and that the more velocity it has, the more it expresses vitality.

-- Rabindranath Tagore

As we begin, we must clearly define what we mean by the term motion.

Motion – the change in a objects position with respect to time

NOTE: Position must always be stated with respect to some stationary object or reference point in order to be meaningful!

(Else, measurements of the same event would yield different results)

What would be some examples of a stationary reference point for a car traveling down the highway?

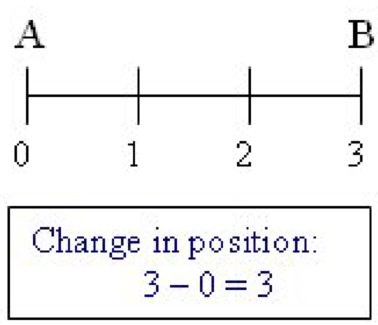
Consistent and accurate measurements require a "**coordinate system**".

Coordinate System – a artificial reference grid imposed on a system in order to make measurements

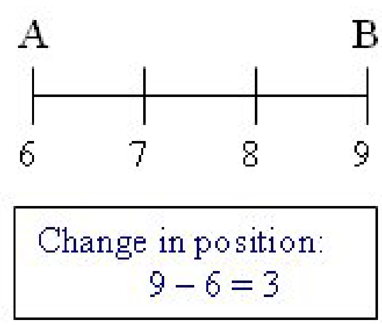
- Our Choice
- No Right or Wrong Way

Ex. An object changing position from **A** to **B**

I



II



Choose the simplest system possible.

Every measurement of motion requires a reference point **AND** a suitable coordinate system.

Reference Frame – a coordinate system that is considered stationary with respect to the object that is in motion.

Motion can be classified into 3 main categories.

1. Translational Motion
2. Rotational Motion
3. Periodic Motion

Translational Motion

- **Linear** (*straight-line or 1 dimensional motion*)

- **Projectile** (*arc'ed motion*)

NOTE: When discussing translational motion, scientists often use the term **TRAJECTORY** to represent the path an object moves along.

Rotational Motion

- **Circular**

- **Non-Circular**

Periodic Motion

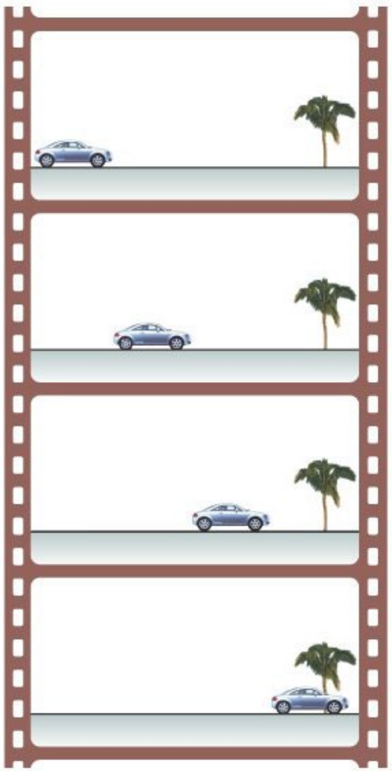
- **Oscillations**

- **Vibrations**

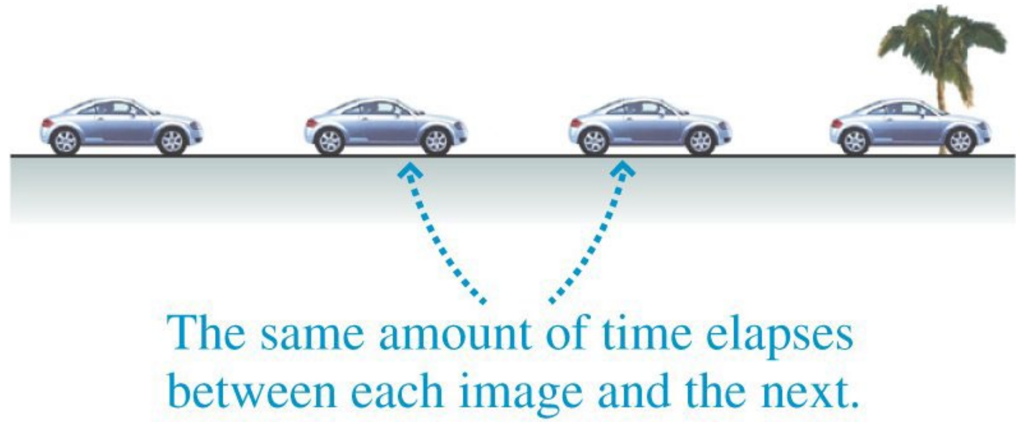
Complex motion is a combination of 2 or more of the basic motion types.



Motion Diagrams



Film Strip View



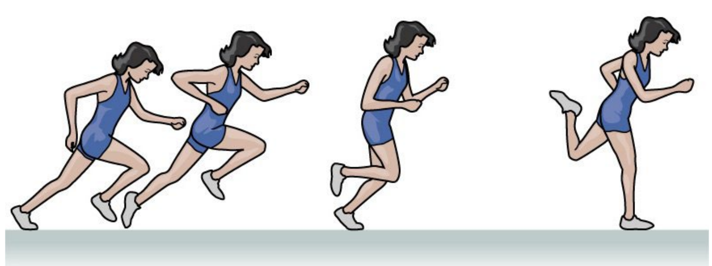
Composite View
(Motion Diagram)

Motion diagrams provide a way to visually interpret the change in an objects motion with respect to time.

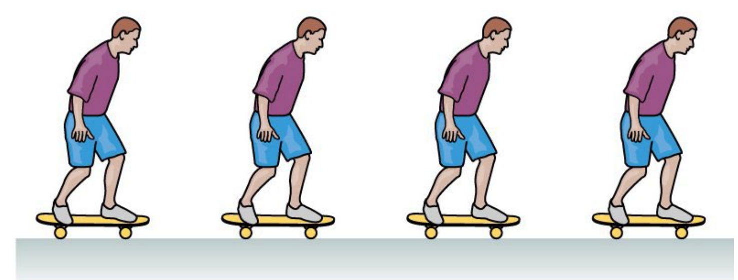
Motion Diagram Examples

No Motion (At Rest)

Rate of Motion is Decreasing



Rate of Motion is Increasing



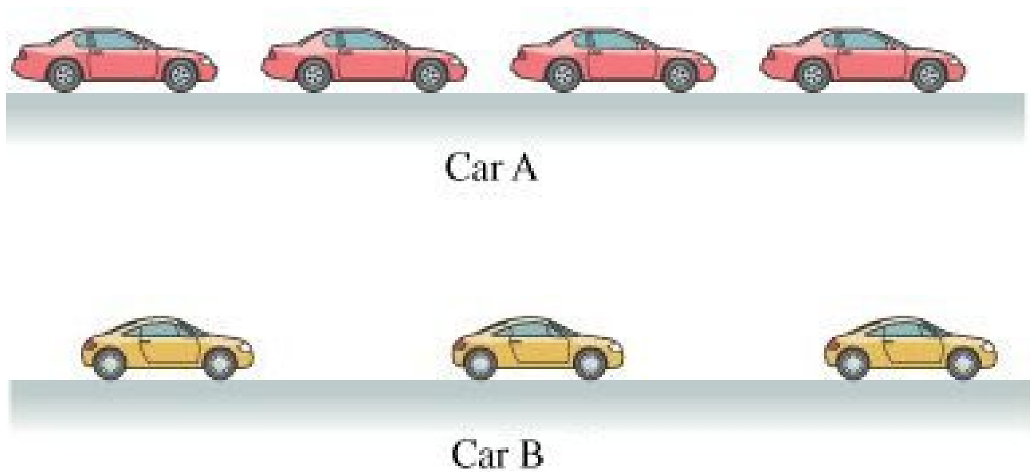
Constant Rate of Motion

Knowledge Inventory

Cars **A** and **B** are traveling at different constant rates of motion.

Which car is going "faster" (*has a higher rate of motion*), **A** or **B**?

(*Justify your reasoning – Assume the time interval is the same for both cars*)

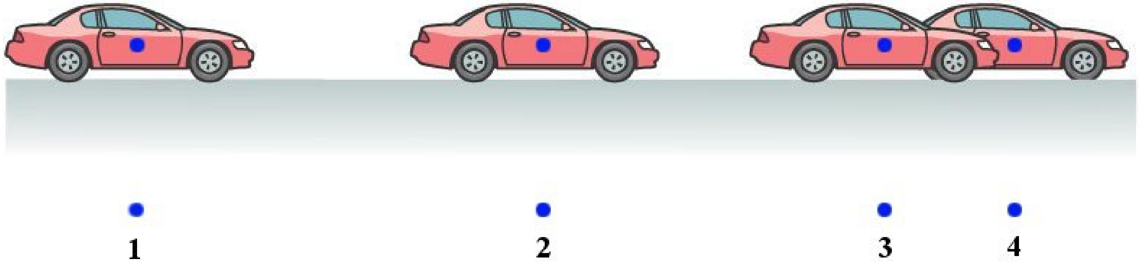


ANSWER: B

Modeling Translational Motion

To describe the motion of a *'solid or rigid'* object, all that is necessary is to track a single, fixed point on the object.

Ex. Modeling the Motion of a Car that is Slowing Down



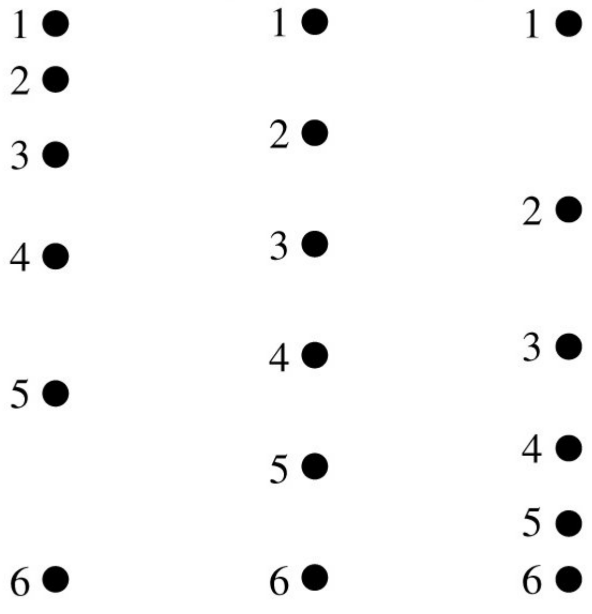
Knowledge Inventory

Match the motion diagram with its possible description.

A: A dust particle settling to the ground at a constant speed

B: A ball dropped from the roof of a building

C: A rocket descending slowly in order to make a soft landing



B **A** **C**

When modeling a rigid object as a single point, we also treat the object as if all of its mass were concentrated at that point. This modeling trick is called the particle model.

Particle Model

A simplification in which the mass of an object is treated as if all of it were concentrated at a single point.

Limitations of the Particle Model

Most effective when describing the translational motion of ‘rigid’ objects.

The Tortoise and the Hare

Race is along a straight 2 mile track

Tortoise (0.2 *mph*)

Runs at a constant rate

Rabbit (15 *mph*)

Runs for 5 min
Plays and sleeps for 9 hr 53 min
Finishes race in 2 min

How far ahead of the rabbit is the tortoise when he crosses the finish line?

1 mile
.75 mile
.5 mile
.25 mile

Linear (Straight Line) Motion – Model #1

The simplest motion model is defined as the total ***distance*** traveled divided by the elapsed transit time. This quantity is called the **average speed** of an object.

➔ $$rate\,of\ linear\ motion = \frac{\text{distance}}{\text{elapsed time}} \equiv \textbf{average speed}$$

The mathematical form of the average speed model is:

$$v_{avg} \equiv \frac{d}{\Delta t}$$

Δ means "change in"

$$\Rightarrow \quad \Delta t = t_{final} - t_{initial}$$

By *choosing* $t_{initial}$ to be zero at the start of **all** time measurements, this expression simplifies to:

$$v_{avg} \equiv \frac{d}{t}$$

The SI units associated with speed are:

$$\frac{\text{distance}}{\text{elapsed time}} = \frac{length}{time} = \frac{m}{s}$$

Speed is always a positive value because distance and time are always positive!

NOTE: *'fast' and 'slow' are relative terms used primarily when comparing one objects rate of motion to another*.

Ex.

A bird travels faster than a snail, but slower than a bullet.

Ex.

A car travels at a constant speed of 55 *mph*. How far will it travel in 3 *hrs*?

NOTE: If the speed is constant, then $v_{constant} = v_{avg}$

$$d = v_{avg}t$$

$$d = \left(55\frac{mi}{hr}\right)(3\,hr)$$

$$d = 165\,mi$$

Ex.

If you travel from Arkadelphia to Little Rock in ¾ *hr* (a distance of ~60 *mi*), what is your average speed in ***m/s***?

Method I

Converting Units at the End

$$v_{avg} = \frac{d}{t}$$

$$v_{avg} = \frac{60\,mi}{.75\,hr} = 80\,mph$$

$$(80\,mph)\left(\frac{.447\frac{m}{s}}{1\,mph}\right) = 35.76\frac{m}{s}$$

Method II

Converting Units Upfront

$$d = (60\,mi)\left(\frac{1609\,m}{1\,mi}\right) = 96540\,m$$

$$t = (.75\,hr)\left(\frac{3600\,s}{1\,hr}\right) = 2700\,s$$

$$v_{avg} = \frac{96540\,m}{2700\,s} = 35.76\frac{m}{s}$$

The Tortoise and the Hare (*Continued*)

- Time it took Tortoise to complete the race:

$$t = \frac{d}{\bar{v}} = \frac{2\ miles}{.2\ mph}$$

$$t = 10\,hr$$

Time required to cover a distance of 2 miles

- Distance the rabbit covers in 10 hrs:

$$d = (15\,mph)(5\min) + (0\,mph)(9\,hr\,53\min) + (15\,mph)(2\min)$$

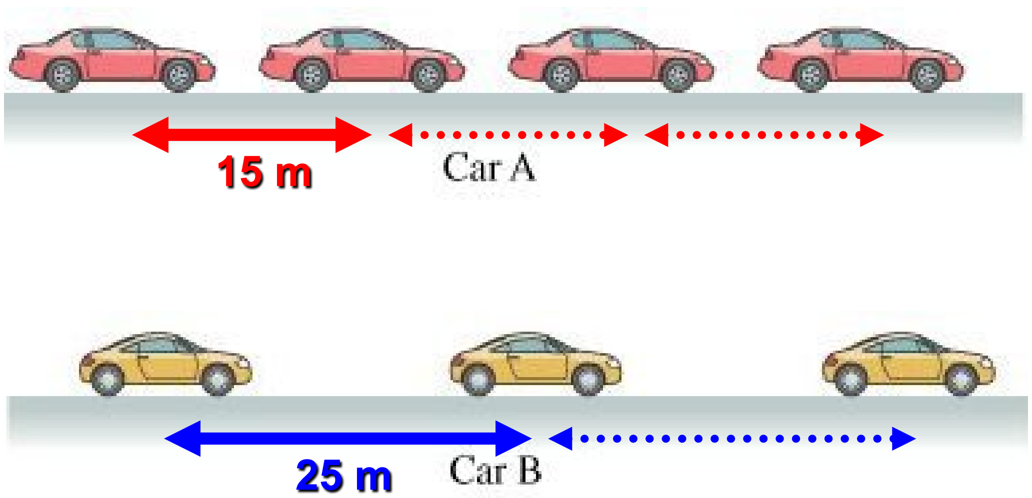
$$d = (15\,mph)\left(\frac{1}{12}hr\right) + (15\,mph)\left(\frac{1}{30}hr\right)$$

$$d = 1.75\,miles$$

The rabbit is a quarter mile (0.25) behind!

Ex.

Previously, we determined that car **B** was traveling "faster" than car **A.** Assuming that the distances shown below were both measured during 2 *s* time intervals, find the average speed of each car to support or discount our previous conclusion.



A

$$v_{avg} = \frac{15\,m}{2\,s} = 7.5\,\tfrac{m}{s}$$

B

$$v_{avg} = \frac{25\,m}{2\,s} = 12.5\,\tfrac{m}{s}$$

B is traveling "faster".

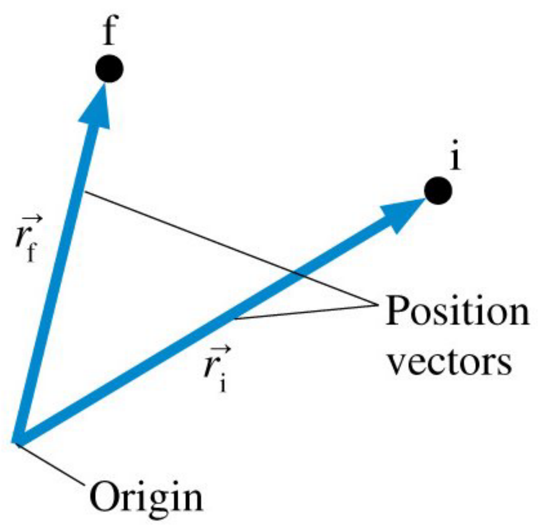
Previously, we stated that **position** could be measured using either *distance* or *displacement*. Our very first rate of motion model used total ***distance*** and the elapsed time to define **average speed**.

Next, we are going define a directional rate of motion model that uses ***displacement*** and elapsed time. This model will be a **VECTOR** model since displacement is a vector!

The **position vector** (**r**)

a vector that specifies the location of an object relative to a fixed reference point (origin).

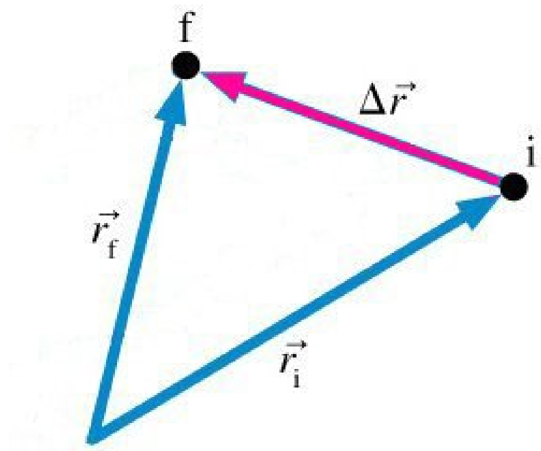
Graphically:



Once the initial and final positions have been specified using position vectors, the **net displacement** (Δ**r**) is:

$$\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$

Graphically:



The directional rate of motion model defined as the ***net displacement*** of an object divided by the elapsed transit time is called the **average velocity**.

$$directional\ rate\ of\ linear\ motion = \frac{\text{net displacement}}{\text{elapsed time}} \equiv \textbf{average velocity}$$

The mathematical form of our new velocity model is:

$$\vec{v}_{avg} \equiv \frac{\Delta\vec{r}}{\Delta t} \quad or \quad \mathbf{v}_{avg} \equiv \frac{\Delta\mathbf{r}}{\Delta t}$$

The SI units that are associated with velocity are:

$$\frac{\text{displacement}}{\text{elapsed time}} = \frac{length}{time} = \frac{m}{s}$$

Warning:

The **units** for **speed** and **velocity** are exactly the same. This results from both models measuring a *change in position* (length) *divided by time*.

Unlike *Speed* which is always a positive, *velocity* can be negative **if** the displacement is in a negative direction [due to the choice of coordinate system].

Since velocity is a vector quantity, we can represent it using vector arrows:

- direction of the motion = direction of the arrow
- magnitude of the velocity = length of the arrow.

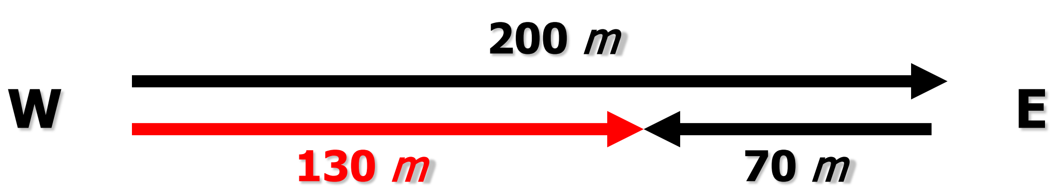
Warning:

To specify a *velocity* numerically, a number **AND** a direction must be given. If no direction is given, the value could easily be misinterpreted as *speed*.

Distinguishing Between Speed & Velocity

Ex. A car travels East 200 *m* in 8 *s* and then returns due West 70 *m* in 5 *s*. What is the *average speed* and *average velocity* of the car?

Visual Model of the Event



Speed:

$$d = \bar{v}t$$

$$d = 200\,m + 70\,m = 270\,m$$

$$t = 8\,s + 5\,s$$

$$\bar{v} = \frac{270m}{13s} = 20.8\frac{m}{s}$$

(~ 46.5 mph)

Velocity (in 1 D):

$$\left(\mathbf{v}_x\right)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{200\,m - 70\,m}{13\,s - 0\,s}$$

$$\left(\mathbf{v}_x\right)_{avg} = \frac{130m}{13s} = 10\frac{m}{s}\ \textbf{East}$$

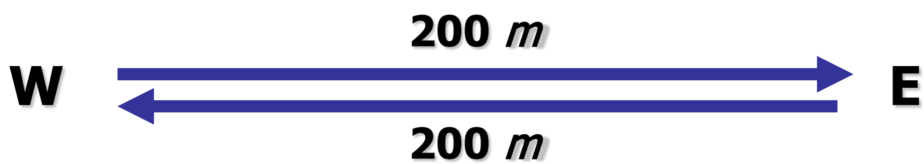
(~ 24.4 mph)

DEMO – Around the Room

- In order to meet at the same place at the same time, the person with the greater distance to cover **MUST** travel at a higher rate of motion (speed)!

Ex. A car makes a round trip of 400 m (East 200 *m* then West 200 *m*) in 16 *s*. What is the *average speed* and *average velocity* of the car?

Visual Model



Speed:

$$\bar{v} = \frac{400m}{16s} = 25\frac{m}{s}$$ **(~ 60 mph)**

Velocity:

$$(\mathbf{v}_x)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{0m}{16s} = 0\frac{m}{s}$$

Ex. A car travels East 200 *m* in 8 *s*. What is the *average speed* and *average velocity* of the car?

Visual Model **W** → **200 *m*** → **E**

Speed:

$$\bar{v} = \frac{200m}{8s} = 25\frac{m}{s}$$

(~ 60 mph)

Velocity:

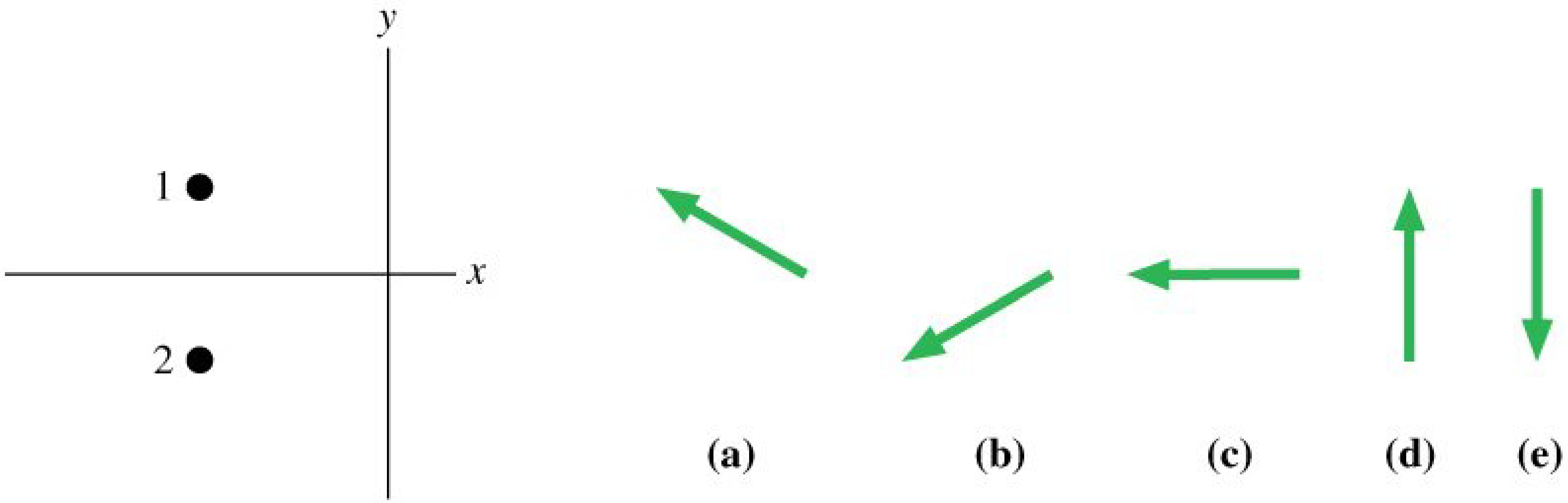
$$(\mathbf{v}_x)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{200m}{8s} = 25\frac{m}{s} \quad \textbf{East}$$

(~ 60 mph)

Speed and velocity can ONLY have the same magnitude if the *distance* and *displacement* have the same magnitude.

Knowledge Inventory

A particle moves from position 1 to position 2 during the time interval Δt. Which vector represents the particles *average velocity* ?



(e)

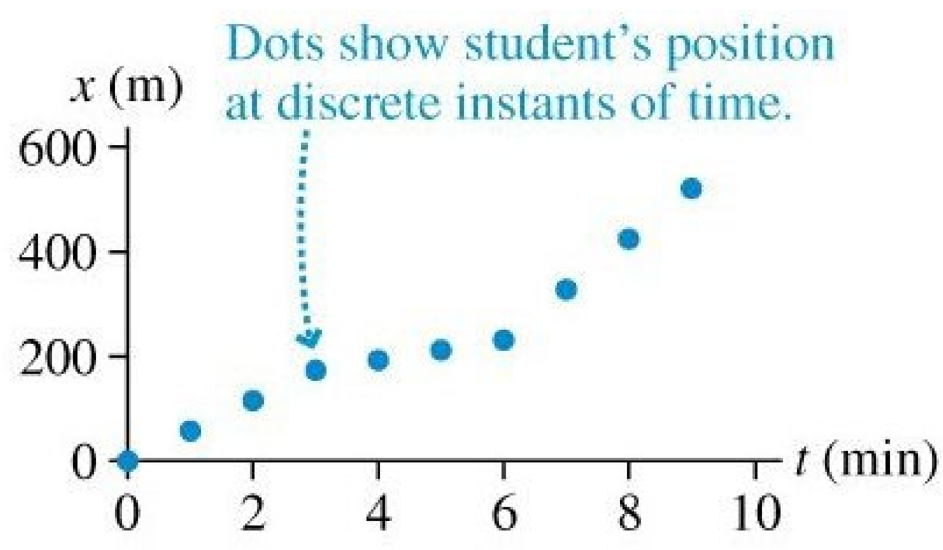
Example

Starting with a coordinate system such that x & $t = 0$ at her house, measurements of a student's position on her way to school are made every minute until she arrives.

Time t (min)	Position x (m)
0	0
1	60
2	120
3	180
4	200
5	220
6	240
7	340
8	440
9	540

Position vs. time graph (Discrete)

- A plot of the data measurements on a position vs. time graph results in a visual representation of where the student was at discrete moments in time (*i.e. when the data was taken*).



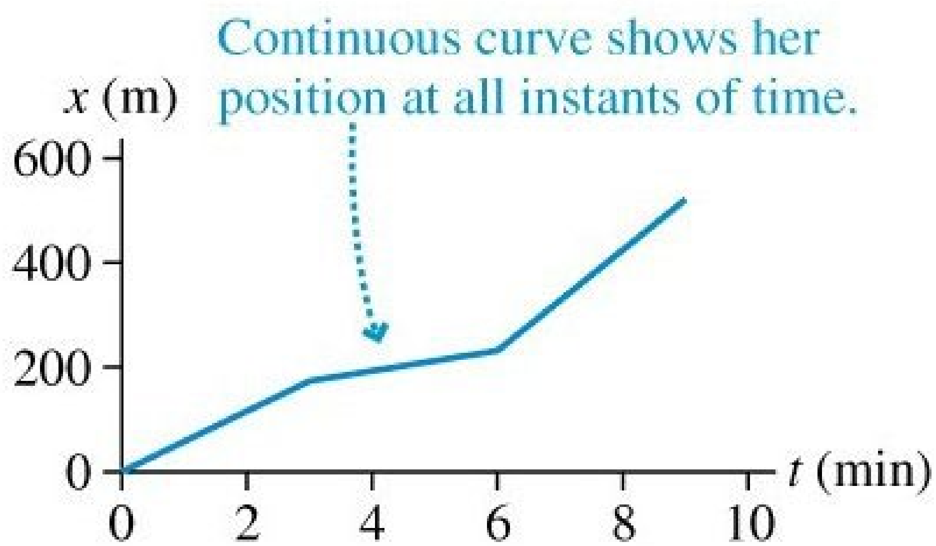
Position vs. time graph (Continuous)

- Common sense tells us that:
  - The student was somewhere specific at all times
  - The student moved continuously through all points in space
  - She never occupied two positions at the same time

Therefore, it is reasonable to assume that her motion **could be** shown as a continuous curve passing through the measured data points.

Position graph – a continuous curve that shows an object's position as a function of time

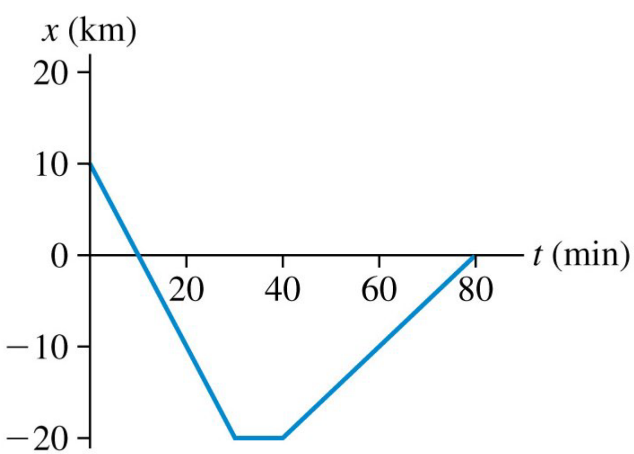
Position Graph

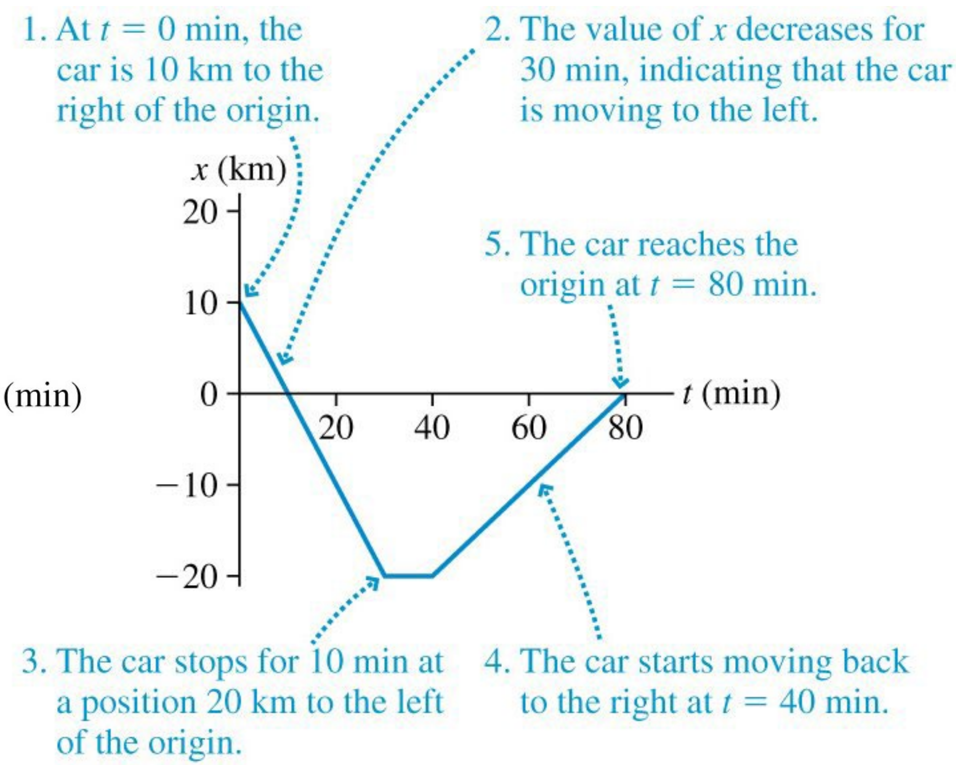


NOTE: A graph is not a "picture" of the motion, but rather an abstract "visual representation" of the motion. For instance, the student walks along a straight line during the motion, but the position graph itself is not a straight line.

Knowledge Inventory

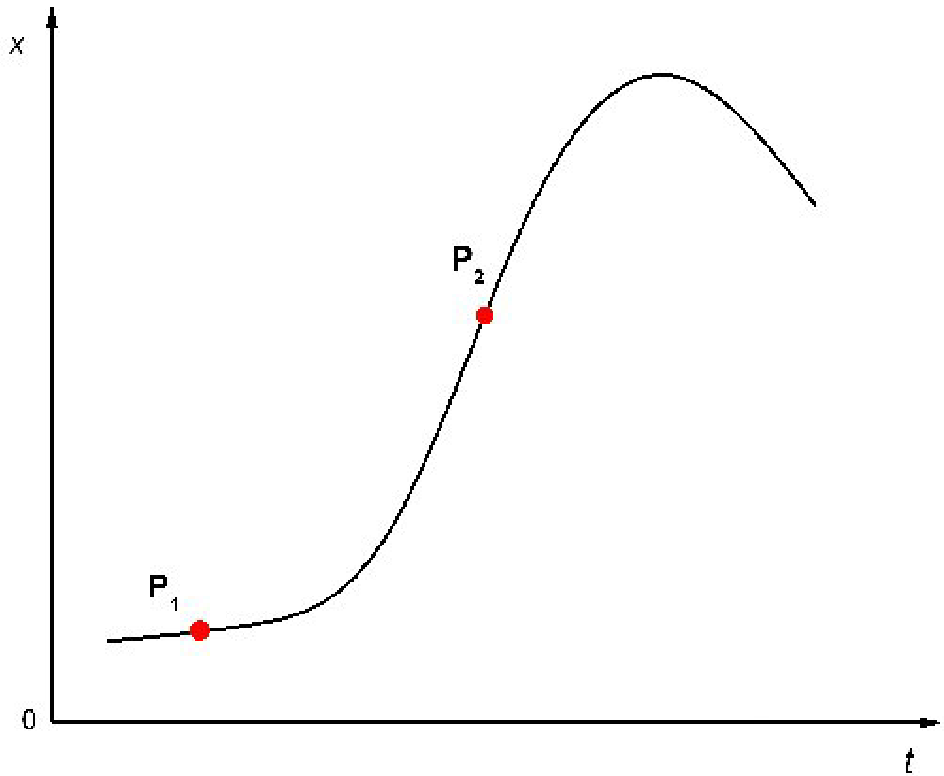
The following graph represents the motion of a car along a straight road. Describe the motion of the car.





Average velocity from a position graph

- Consider the following *x vs t* position graph that depicts the motion of a particle in 1 Dimension.



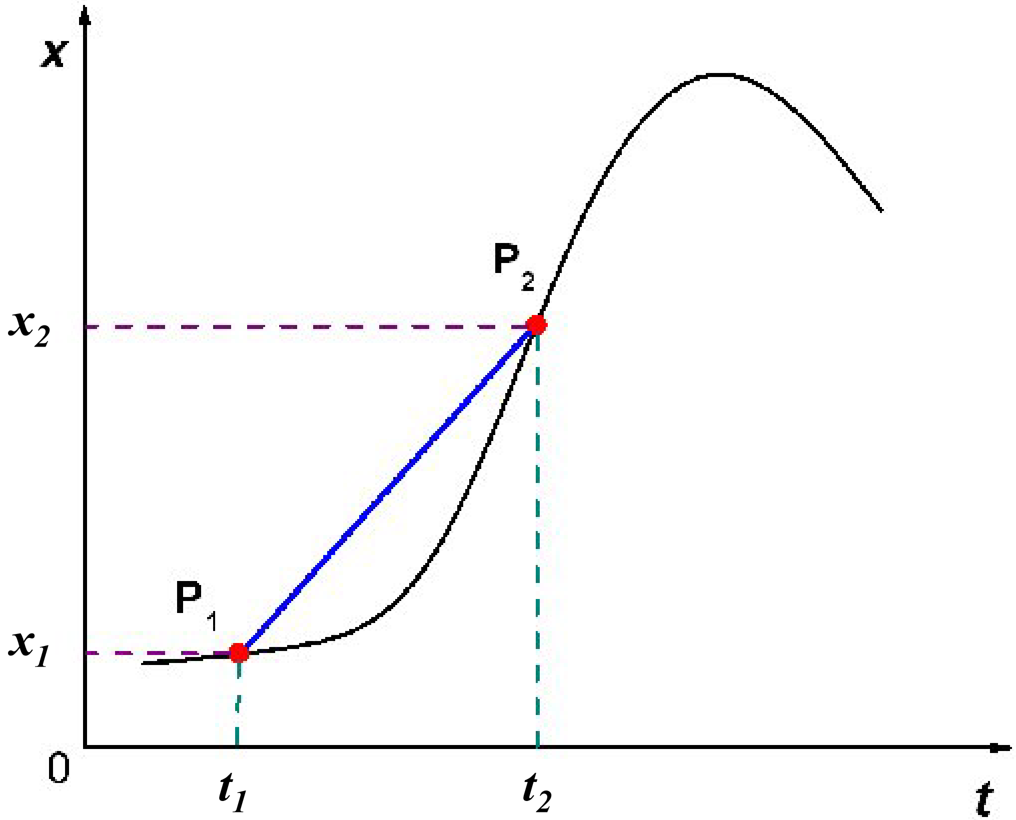
What is the average velocity between P_1 and P_2?

By definition,

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

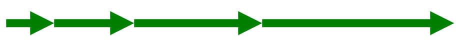
$$in\ 1D : (\mathbf{v}_x)_{avg} = \frac{\Delta \mathbf{x}}{\Delta t}$$

Average velocity *Cont* ...

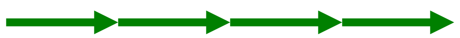


Analyzing the Motion

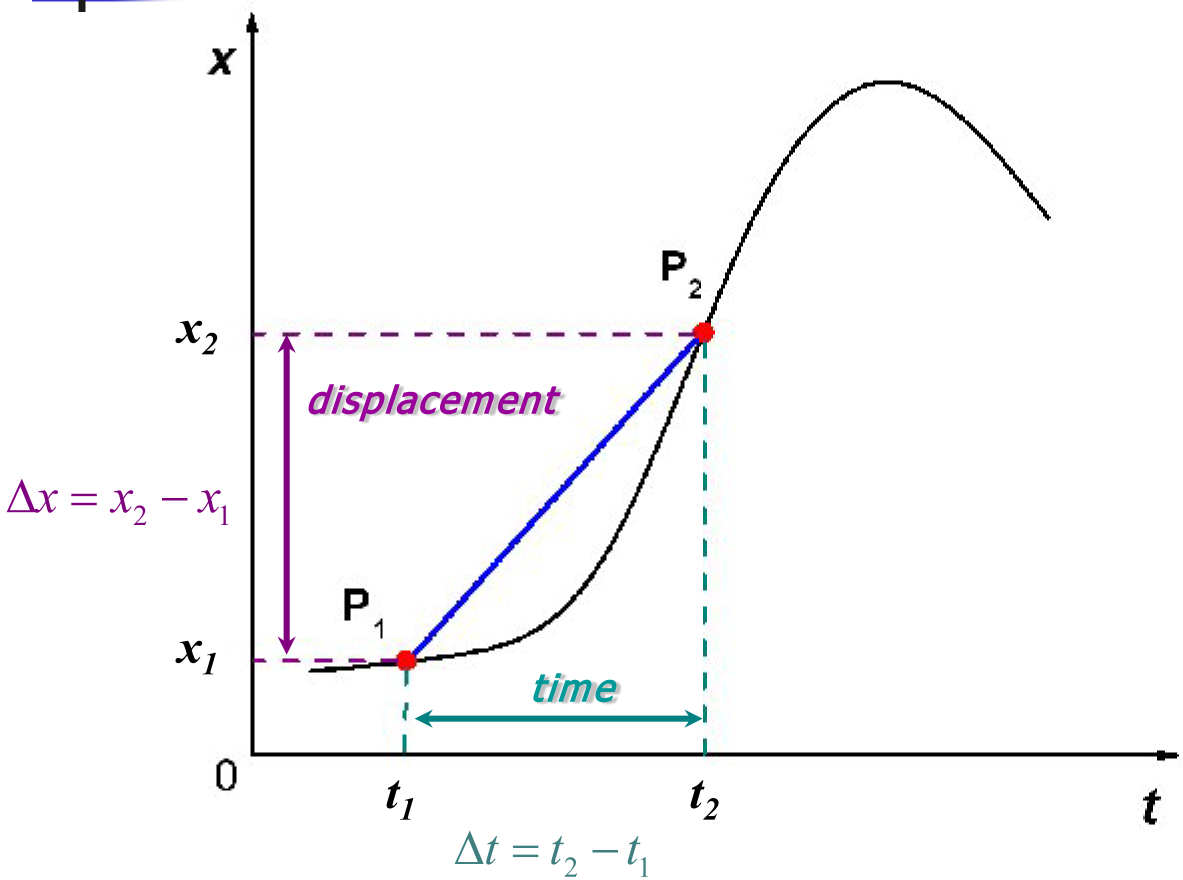
The motion between P_1 & P_2 is **NOT** steady. The position (*black line*) changes more rapidly as time increases.



However, motion along the *blue line segment* **WOULD BE** steady since equal displacements would be covered in equal times.



Average velocity *Cont* ...



Recall: The slope of a line is the ratio of the change in the *y-axis* over the change in the *x-axis*.

$$slope = \frac{\Delta y}{\Delta x}$$

On a position vs. time graph, the ***average velocity*** would be equal to the slope of the ***blue line*** through points P_1 & P_2.

$$slope = \frac{\Delta x}{\Delta t} \equiv \left(v_x\right)_{avg}$$

Pitfall Prevention

- In any graph of physical data, the axes will have units associated with them.

➔ **The slope will also have units!**

Ex. Position vs. Time Graph

slope = position / time

position has units of length (m)
time has units of time (s)

➔ the slope will have units of m/s!

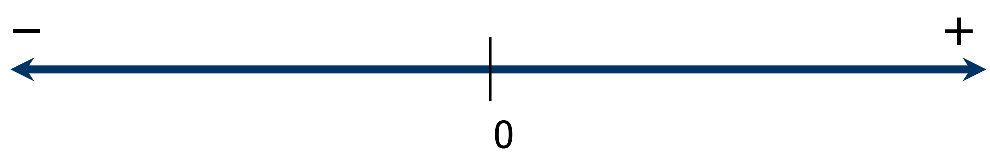
Uniform Motion

- Uniform motion can be defined in several ways:
  - Motion which covers equal displacements in equal intervals of time
  - Motion in which an objects velocity is constant

    (*this implies that the magnitude and direction do* ***NOT*** *change* → straight-line motion at a constant rate)

Modeling 1 Dimensional Uniform Motion

- Since uniform motion is motion in a straight line, it is easily modeled by 1 Dimensional quantities.
- For motion in 1 Dimension, vectors are restricted to point only "forward" or "backward".
- A suitable reference system to quickly and easily indicate a "forward" or "backward" vector quantity would be a single axis with one direction positive and the other negative.

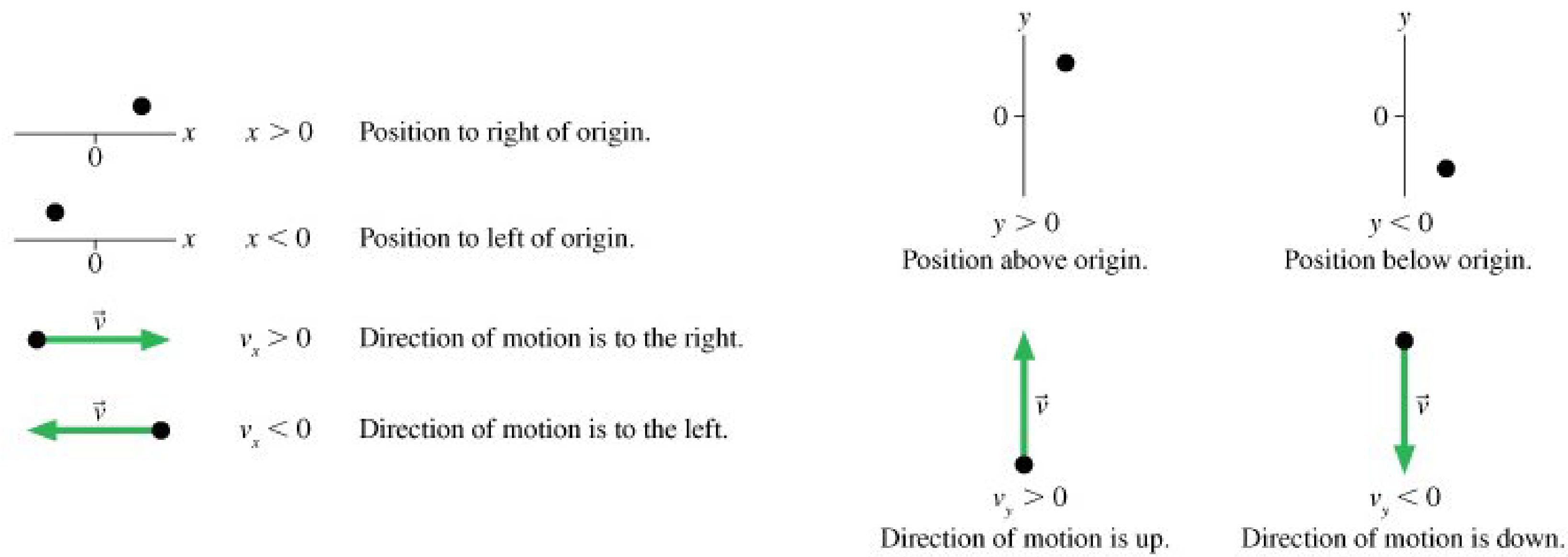


A Comment on Notation

- When dealing with uniform motion, there is no real need to specify "***average***" for velocity since the value of the velocity is constant at all times.

- Therefore, we will **drop** the avg subscript and refer to an *average velocity* simply as v_x or v_y.

1 Dimensional Sign Convention



- The sign of position (x or y) indicates ***where*** an object is relative to the origin
- The sign of velocity (v_x or v_y) indicates ***which direction*** the object is moving relative to the origin

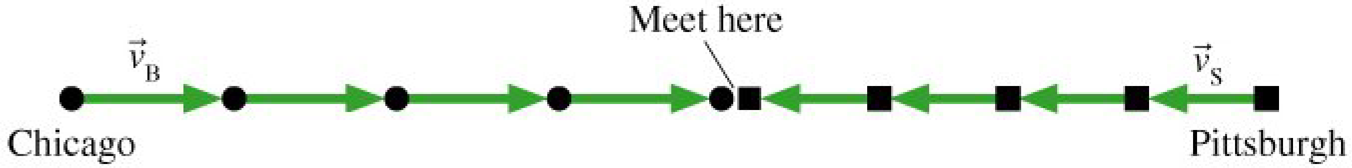
Using Position Graphs to Solve Problems

- It is possible to use position graphs to solve certain types of motion problems.

Example

- Bob leaves home in Chicago at 9 *am* and travels East at a steady 60 *mph*. Susan, 400 *miles* to the east in Pittsburg, leaves at the same time and travels West at a steady 40 *mph*. Where will the two travelers meet?

Motion Diagram Model



Example *Cont*.

- Since their velocities (*both magnitude and direction*) are known, we can construct a position graph for each traveler.

- Because the motion is uniform, the position graphs will be lines. We can use the equation of a line ($y = mx + b$) and the information given in the problem to determine their **position functions** (positions as a function of time).

Bob

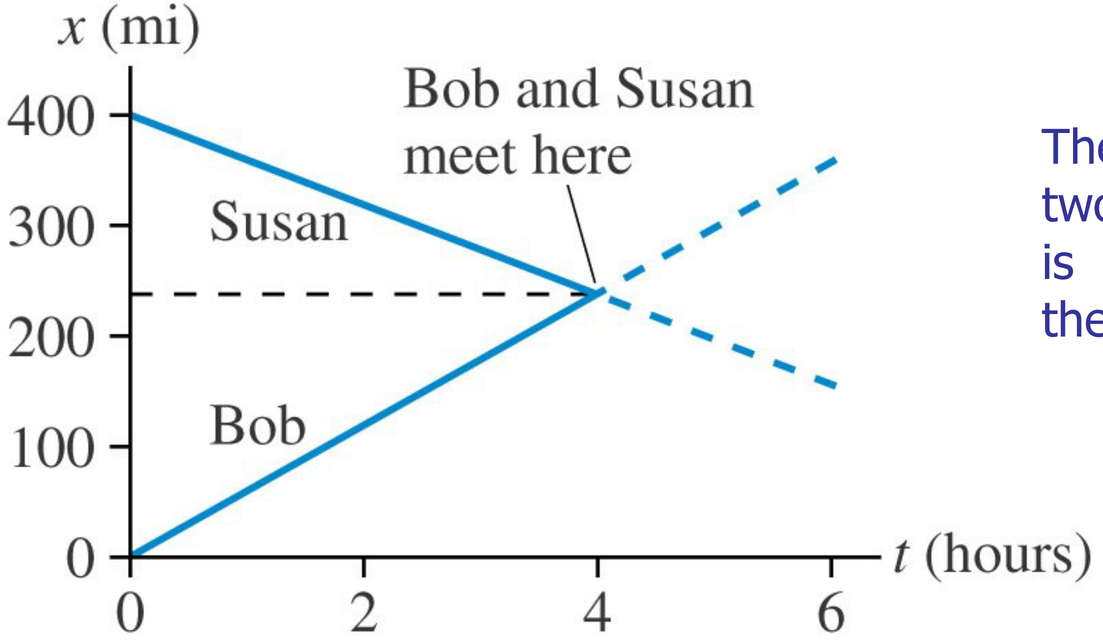
$$x_B(t) = 60t$$

Susan

$$x_S(t) = -40t + 400$$

Example *Cont.*

Plotting both position functions on the same graph yields:



The intersection of the two position functions is the location where they will meet.

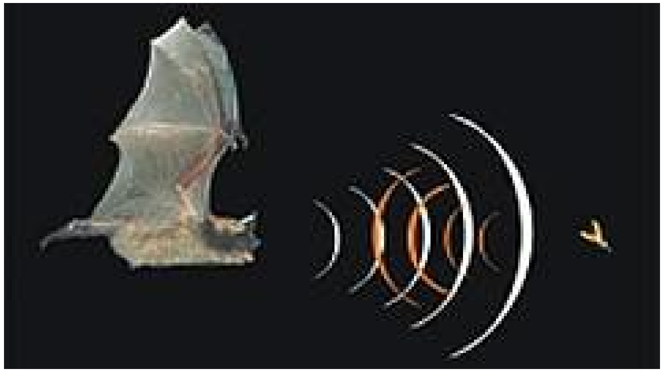
Solving the two linear equations with two unknowns yields:

$x = 240\ miles\ \textbf{East}\ of\ Chicago$

$t = 4\ hrs$

Classroom Exercise

Bats use echolocation to locate prey. Suppose a bat is traveling in a straight line at a constant speed of 19.5 *m/s* toward a vertical wall. If it makes a single clicking sound and hears the echo 0.15 *sec* later, how close is the bat to the wall when it receives its echo?



24.3 m

Assume the speed of sound is 343 m/s.