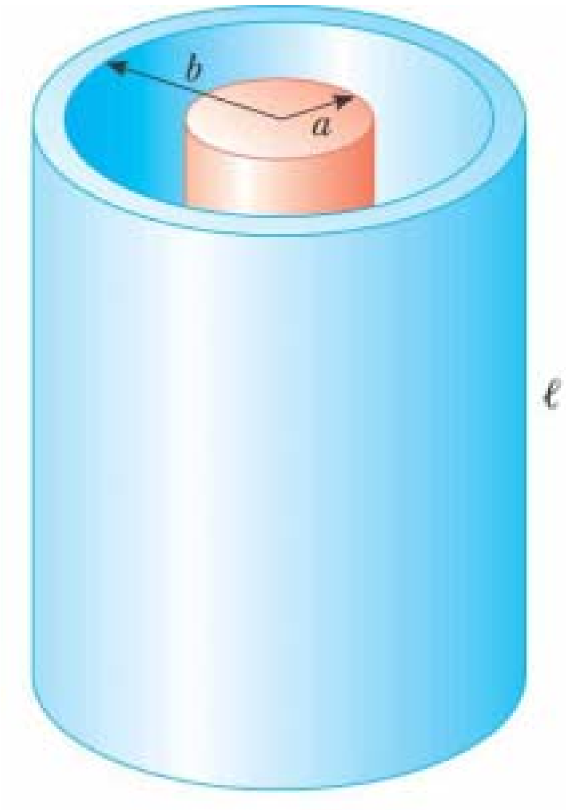
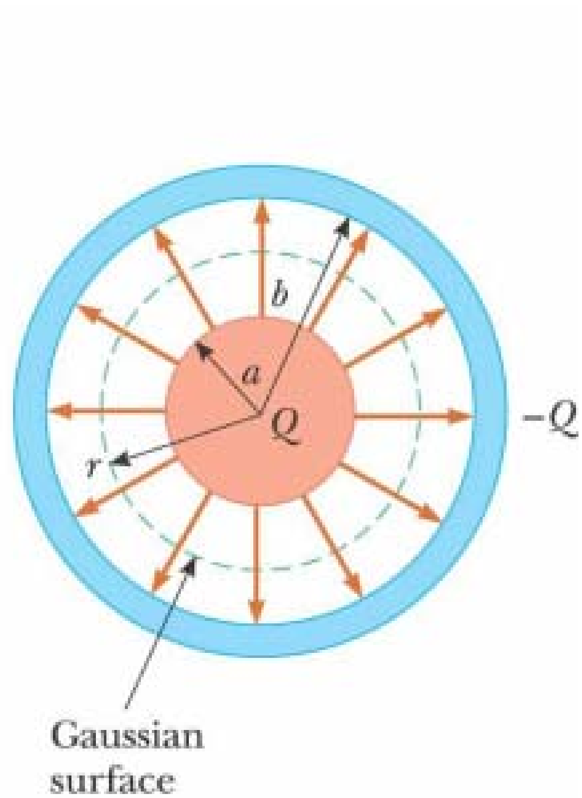
Capacitance of a Cylindrical Capacitor





Find C:

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\Delta V = -\int_a^b \frac{2k_eQ}{rl}\,\hat{\mathbf{r}}\cdot d\vec{r}$$

$$= -\frac{2k_eQ}{l}\int_a^b \frac{dr}{r}$$

$$= -\frac{2k_eQ}{l}\left(\ln r\right)\Big|_a^b = -\frac{2k_eQ}{l}\left(\ln b - \ln a\right)$$

$$\Delta V = -\frac{2k_eQ}{l}\ln\left(\frac{b}{a}\right)$$

Use Gauss's Law to Find **E:**

$$\int \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_o}$$

$$E\int dA = \frac{Q}{\varepsilon_o}$$

$$E\left(2\pi rl\right) = \frac{Q}{\varepsilon_o}$$

$$\vec{E} = \frac{Q}{2\pi\varepsilon_o rl}\hat{\mathbf{r}} \qquad or \qquad \vec{E} = \frac{2k_eQ}{rl}\hat{\mathbf{r}}$$

Since C is $\frac{\Delta Q}{\Delta V}$ and is always a positive value, then

$$C = \frac{|\Delta Q|}{|\Delta V|} = \frac{Q}{\frac{2k_eQ}{l}\ln\left(\frac{b}{a}\right)} \quad \rightarrow \quad C = \frac{l}{2k_e\ln\left(\frac{b}{a}\right)}$$