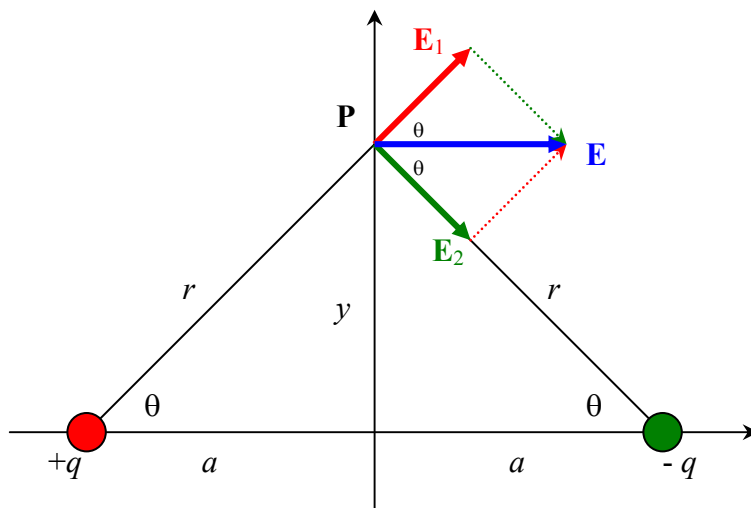


Ex. **Electric Dipole** : A $+q$ & $-q$ separated by a small distance



Charges of a dipole are such that $|+q| = |-q| = q$

$$r^2 = a^2 + y^2$$

At point P (anywhere along the y axis):

$$\mathbf{E}_1 = \frac{k_e |q|}{r_1^2} \hat{\mathbf{r}} = \left(\frac{k_e q}{(a^2 + y^2)} \cos \theta \right) \hat{\mathbf{x}} + \left(\frac{k_e q}{(a^2 + y^2)} \sin \theta \right) \hat{\mathbf{y}}$$

$$\mathbf{E}_2 = \frac{k_e |-q|}{r_2^2} \hat{\mathbf{r}} = \left(\frac{k_e q}{(a^2 + y^2)} \cos \theta \right) \hat{\mathbf{x}} + \left(\frac{k_e q}{(a^2 + y^2)} (-\sin \theta) \right) \hat{\mathbf{y}}$$

The total electric field \mathbf{E} can be found by using the superposition principle:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$\mathbf{E} = \left(\frac{2k_e q}{(a^2 + y^2)} \cos \theta \right) \hat{\mathbf{x}}$$

Notice the y components cancel out since they are equal but opposite in direction.

Since

$$\cos \theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + y^2}}$$

$$\rightarrow \mathbf{E} = \left(\frac{2ak_e q}{(a^2 + y^2)^{3/2}} \right) \hat{\mathbf{x}}$$

NOTE: In the limiting case where $y \gg a$, \mathbf{E} reduces to

$$\mathbf{E} \approx \left(\frac{2ak_e q}{y^3} \right) \hat{\mathbf{x}}$$