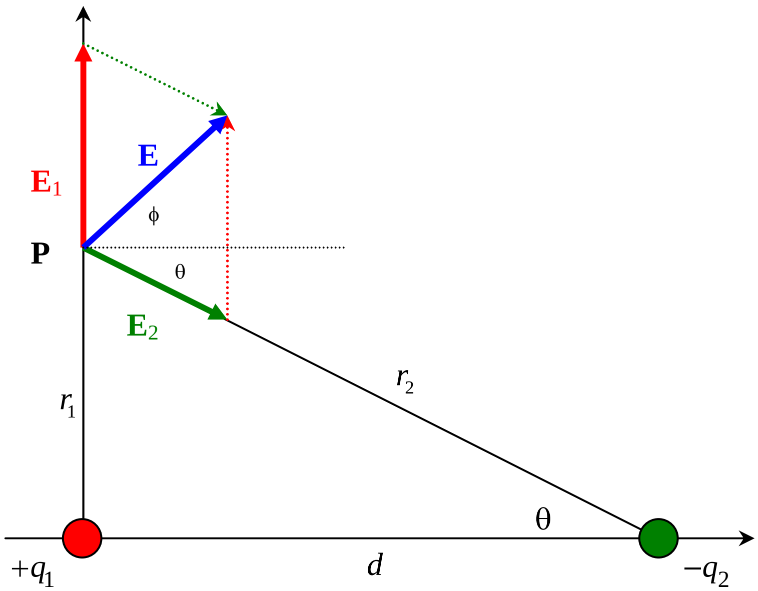
Ex. **Electric Dipole** : A $+q$ & $-q$ separated by a small distance



$$r_2^{\,2} = r_1^{\,2} + d^2$$

$$\cos\theta = \frac{d}{r_2} = \frac{d}{\sqrt{r_1^{\,2} + d^2}}$$

$$\sin\theta = \frac{r_1}{r_2} = \frac{r_1}{\sqrt{r_1^{\,2} + d^2}}$$

At point P (*anywhere along the +y axis through charge +q*):

$$\mathbf{E}_1 = \frac{k_e|+q_1|}{r_1^{\,2}}\hat{\mathbf{r}}_1 = \left(\frac{k_e|q_1|}{r_1^{\,2}}\right)\hat{\mathbf{y}}$$

$$\mathbf{E}_2 = \frac{k_e|-q_2|}{r_2^{\,2}}\hat{\mathbf{r}}_2 = \left(\frac{k_e|q_2|}{r_2^{\,2}}\cos\theta\right)\hat{\mathbf{x}} + \left(\frac{k_e|q_2|}{r_2^{\,2}}(-\sin\theta)\right)\hat{\mathbf{y}}$$

$$\hat{\mathbf{r}}_1 = \hat{\mathbf{y}}$$

$$\hat{\mathbf{r}}_2 = \cos\theta\,\hat{\mathbf{x}} + (-\sin\theta)\hat{\mathbf{y}}$$

NOTE: The sign on cos & sin are determined by quadrant

The total electric field **E** at point **P** can be found by using the superposition principle:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$\mathbf{E} = \left(\frac{k_e|q_2|}{r_2^{\,2}}\cos\theta\right)\hat{\mathbf{x}} + \left(\frac{k_e|q_1|}{r_1^{\,2}} - \frac{k_e|q_2|}{r_2^{\,2}}(\sin\theta)\right)\hat{\mathbf{y}}$$

Notice the y components do NOT cancel out since they are un-equal.

Since

$$\cos\theta = \frac{d}{r_2} = \frac{d}{\sqrt{r_1^{\,2} + d^2}} \quad \& \quad \sin\theta = \frac{r_1}{r_2} = \frac{r_1}{\sqrt{r_1^{\,2} + d^2}}$$

$$\rightarrow \quad \mathbf{E} = \left(\frac{k_e d|q_2|}{\left(r_1^{\,2} + d^2\right)^{3/2}}\right)\hat{\mathbf{x}} + \left(\frac{k_e|q_1|}{r_1^{\,2}} - \frac{k_e r_1|q_2|}{\left(r_1^{\,2} + d^2\right)^{3/2}}\right)\hat{\mathbf{y}}$$

If we are interested in determine the angle **E** makes with respect to the x-axis (ϕ):

$$\phi = \tan^{-1}\left(\frac{E_y}{E_x}\right)$$

$$\phi = \tan^{-1}\left(\frac{|q_1|(r_1^2 + d^2)^{3/2}}{dr_1^2|q_2|} - \frac{r_1}{d}\right)$$ **"reduced"**

NOTE: In the limiting case where $r_1 >> d$, **E** reduces to

$$\mathbf{E} = \left(\frac{k_e d |q_2|}{r_1^3}\right)\hat{\mathbf{x}} + \left(\frac{k_e}{r_1^2}\left(|q_1| - |q_2|\right)\right)\hat{\mathbf{y}}$$

NOTE: For a point sufficiently far away, the direction **E** points in the y direction will only depend on the difference in the charge magnitudes (*since* r_1 *is squared –> always positive*).

$|q_1| > |q_2|$ ➔ $+\hat{\mathbf{y}}$ (*points away from dipole*)

$|q_1| = |q_2|$* ➔ 0

$|q_1| < |q_2|$ ➔ $-\hat{\mathbf{y}}$ (*points toward dipole*)

***** If we have 2 charges with equal magnitudes, the second term vanishes and we are left with an expression for **E** that is exactly like that for an electric dipole with the y-axis half way between the charges.