Flux Through Half a Sphere

A point charge $Q$ is located just above the center of the flat face of a hemisphere of radius $R$ as shown in following Figure. What is the electric flux ($\Phi_E$) due to the point charge

(a) Through the curved part of the surface?
(b) Through the flat face?

![Gaussian Surface (sphere)](image)

$a)$ Since no charge is enclosed by the closed surface, the total flux must be zero.

$$\Phi_{E_{\text{total}}} = \Phi_{E_{\text{flat}}} + \Phi_{E_{\text{curve}}} = 0$$

$$\Rightarrow \quad \Phi_{E_{\text{flat}}} = -\Phi_{E_{\text{curve}}}$$

For a complete sphere that would enclose charge $Q$, the total flux would be:

$$\Phi_{E_{\text{total}}} = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Since only half of the sphere is used and the charge is located at about the center ($\delta \rightarrow 0$), the flux through the bottom surface is just half the value for the total sphere:

$$\Phi_{E_{\text{curve}}} = \frac{Q}{2\varepsilon_0} \quad \text{Note: This is a positive value indicating that the Electric field lines are leaving the surface}$$

$$b) \quad \Phi_{E_{\text{flat}}} = -\frac{Q}{2\varepsilon_0} \quad \text{Note: This is a negative value indicating that the Electric field lines are entering the surface}$$