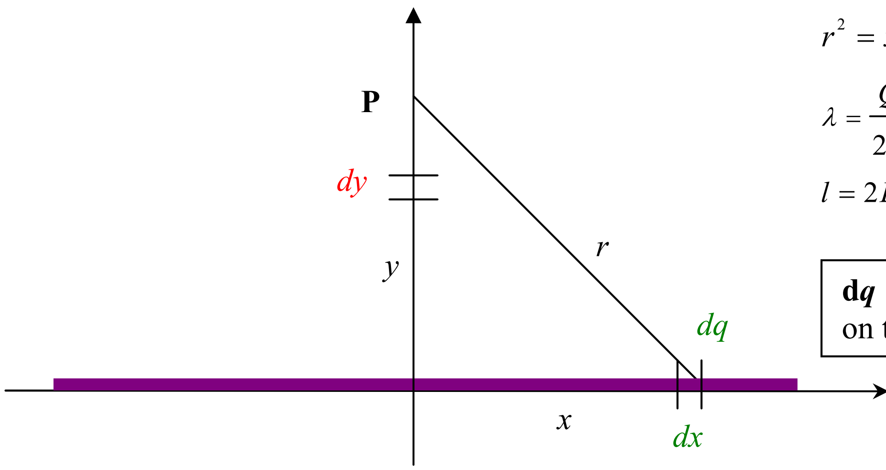
Ex. Electric Potential due to a Finite Charged Rod

Find the electric potential some distance y above a uniformly charged finite rod



$r^2 = x^2 + y^2$

$\lambda = \dfrac{Q}{2L} \Rightarrow$ Total Linear Charge Density

$l = 2L \Rightarrow$ Total Length of the Rod

dq represents the amount of charge on the rod piece of length **dx**.

Since this is a uniform charged rod → $\lambda = \dfrac{dq}{dx}$ ➔ $dq = \lambda dx$

At point P:

$$V = k_e \int \frac{dq}{r}$$

$$V = k_e \int_{-L}^{L} \frac{\lambda dx}{\sqrt{x^2 + y^2}}$$

Since x^2 is an even function, we can change the limits of integration from $-L$ to L to 2 times the integral from 0 to L.

$$V = 2\lambda k_e \int_{0}^{L} \frac{dx}{\sqrt{x^2 + y^2}}$$

From integration tables:

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

➔ $$V = 2\lambda k_e \left[\ln\left(L + \sqrt{L^2 + y^2}\right) - \ln(y) \right]$$

$$\mathbf{V} = \frac{k_e Q}{L} \ln\left(\frac{L + \sqrt{L^2 + y^2}}{y} \right)$$

Using $\lambda = \dfrac{Q}{2L}$ & $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$

Alternate Integration:

We can also determine the electric potential by using the electric field for a finite charged rod.

Recall:

$$\mathbf{E} = \frac{2k_e \lambda L}{y\sqrt{y^2 + L^2}}\hat{\mathbf{y}} \qquad \textit{for a finite charge rod}$$

Using $\mathrm{V} = -\int_{\infty}^{y} \mathbf{E} \cdot d\vec{l}$ **Remember**: Our V = 0 point is at $r = \infty$ (or $y = \infty$ in this case).

$$\mathrm{V} = -\int_{\infty}^{y} \frac{2k_e \lambda L}{y'\sqrt{y'^2 + L^2}}\hat{\mathbf{y}} \cdot d\vec{y}\,'$$

$$\mathrm{V} = -2k_e \lambda L \int_{\infty}^{y} \frac{dy'}{y'\sqrt{y'^2 + L^2}}$$

Note: $\hat{\mathbf{y}} \cdot d\vec{y} = |\hat{\mathbf{y}}||dy'|\cos\theta = dy'$, since $\theta = 0$.

From integration tables:

$$\int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right)$$

➔ $$\mathrm{V} = -2k_e \lambda L \left[-\frac{1}{L}\ln\left(\frac{L + \sqrt{L^2 + y^2}}{y}\right) + \ln(1) \right]$$

$$\mathrm{V} = 2k_e \lambda \ln\left(\frac{L + \sqrt{L^2 + y^2}}{y}\right)$$

Note: $\ln(1) = 0$

$$\mathrm{V} = \frac{k_e Q}{L}\ln\left(\frac{L + \sqrt{L^2 + y^2}}{y}\right)$$

Using $\lambda = \frac{Q}{2L}$

Which is the same result we obtained before.