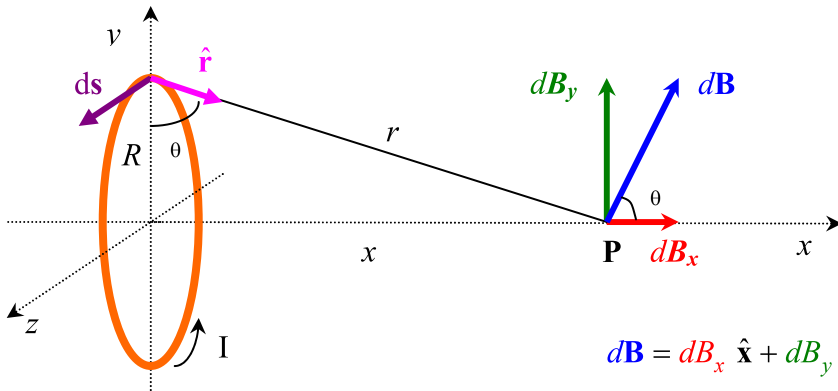
*Ex.* The magnetic field of a *Circular Current Loop* along the central axis




$$r = \sqrt{x^2 + R^2}$$

$$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{x^2 + R^2}}$$

$$d\mathbf{B} = dB_x\,\hat{\mathbf{x}} + dB_y\,\hat{\mathbf{y}}$$

**** NOTE: $d\mathbf{B}$ is in the direction of $d\mathbf{s} \times \hat{\mathbf{r}}$ .**

**** From Symmetry, all the *y* components to cancel out leaving only a magnetic field component along the *x*-axis.**

$$dB_x = dB\cos\theta$$

Using:

$$dB = \frac{\mu_o I}{4\pi}\left(\frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}\right) \ \& \ \left|d\mathbf{s} \times \hat{\mathbf{r}}\right| = ds\,,$$ we can write $dB_x$ as:

$$dB_x = \left(\frac{\mu_o I}{4\pi}\right)\left(\frac{\cos\theta\, ds}{x^2 + R^2}\right)$$

Using:

$\cos\theta = \dfrac{R}{r}$ and integrating around the path, we get

$$B_x = \left(\frac{\mu_o I R}{4\pi\left(x^2 + R^2\right)^{3/2}}\right)\oint ds$$

Since the path is a circle, the path integral around the loop is just the circumference of a circle:

$$B_x = \left(\frac{\mu_o I R}{4\pi\left(x^2 + R^2\right)^{3/2}}\right)(2\pi R)$$

$$B_x = \frac{\mu_o I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

**Using:** $\mathbf{B} = B_x\ \hat{\mathbf{x}} + B_y\ \hat{\mathbf{y}}$

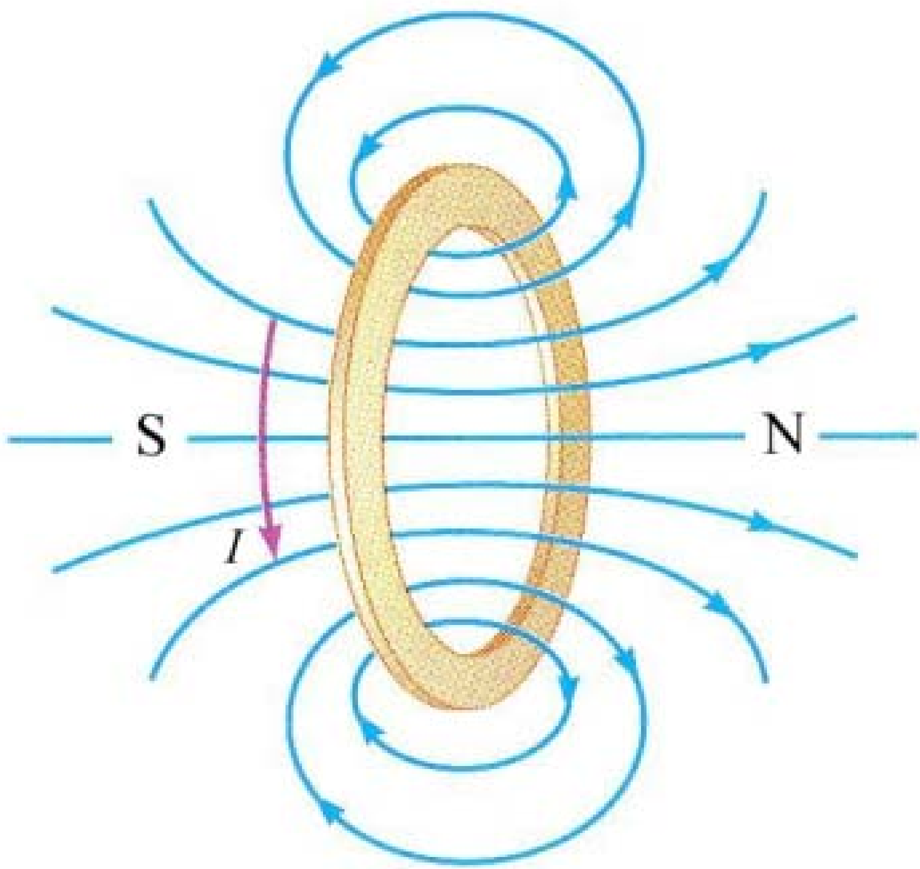
$$\mathbf{B} = \frac{\mu_o I R^2}{2\left(x^2 + R^2\right)^{3/2}}\ \hat{\mathbf{x}}$$

**Limiting Cases of B:**

$$x = 0: \qquad \mathbf{B} = \frac{\mu_o I}{2R}\ \hat{\mathbf{x}}$$

$$x >> R: \qquad \mathbf{B} = \frac{\mu_o I R^2}{2x^3}\ \hat{\mathbf{x}}$$

*** What do the magnetic field lines look like for the *entire* circular current loop?**




**Note: If the current were reversed, the Field lines would point in the other direction (*switching the* N *and* S *pole locations*).**