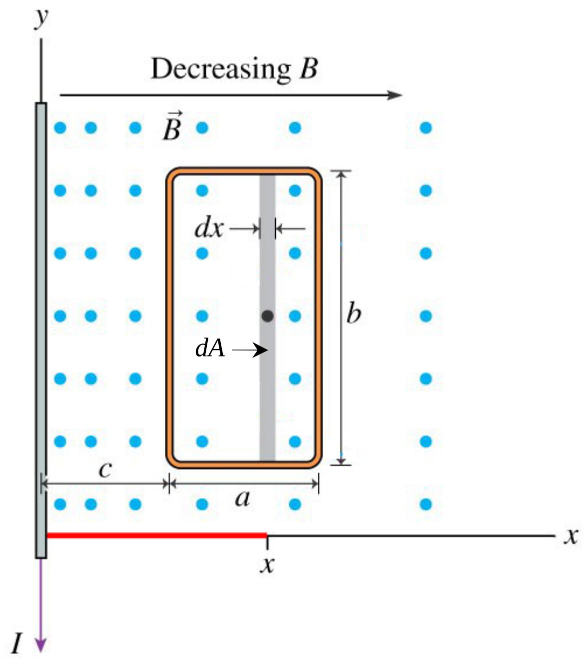
Ex. Magnetic flux thru a rectangular loop due to a nearby wire



$d\mathbf{A} = dA\,\hat{\mathbf{z}}$ (*out of the page*)

B for a wire a distance x away is:

$$\mathbf{B} = \frac{\mu_o I}{2\pi x}\hat{\mathbf{z}} \quad (\textit{out of the page})$$

Using our expression for Magnetic Flux

$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$$

$$\Phi_B = \int \left(\frac{\mu_o I}{2\pi x}\hat{\mathbf{z}}\right)\cdot\left(dA\,\hat{\mathbf{z}}\right)$$

$$\Phi_B = \int \frac{\mu_o I}{2\pi x}dA\ \left(\hat{\mathbf{z}}\cdot\hat{\mathbf{z}}\right) \quad (\hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1)$$

But, $dA = b\,dx$

$$\Phi_B = \int_c^{a+c} \frac{\mu_o I b}{2\pi x}dx$$

$$\Phi_B = \frac{\mu_o I b}{2\pi}\int_c^{a+c} \frac{dx}{x}$$

$$\Phi_B = \frac{\mu_o I b}{2\pi}\left(\ln x\Big|_c^{a+c}\right)$$

Simplifying yields:

$$\Phi_B = \frac{\mu_o I b}{2\pi}\ \ln\left(1+\frac{a}{c}\right)$$