Visual Representation of the Dot Product (*Scalar Product*)

This shows that the dot product is the amount of $\mathbf{A}$ in the direction of $\mathbf{B}$ times the magnitude of $\mathbf{B}$. This is extremely useful if you are interested in finding out how much of one vector is projected onto another or how similar 2 vectors are in direction. The following 5 cases summarize the possible interpretations of the dot product.

**CASE I**  
$\mathbf{A} \cdot \mathbf{B} = AB$  
The interpretation is that all of $\mathbf{A}$ is projected onto $\mathbf{B}$  
(both $\mathbf{A}$ and $\mathbf{B}$ are in the same direction - *parallel*)

\[
\theta = 0 \quad \rightarrow \quad \cos \theta = 1
\]

**CASE II**  
$\mathbf{A} \cdot \mathbf{B} = C$  
$0 < C < AB$  
The interpretation is some of $\mathbf{A}$ is projected onto $\mathbf{B}$  
($\mathbf{A}$ and $\mathbf{B}$ point in the same general direction, *how much depends on the value of C*)

\[
\theta < \frac{\pi}{2} \quad \rightarrow \quad 0 < \cos \theta < 1
\]

**CASE III**  
$\mathbf{A} \cdot \mathbf{B} = 0$  
The interpretation is that none of $\mathbf{A}$ is projected onto $\mathbf{B}$  
($\mathbf{A}$ and $\mathbf{B}$ are *perpendicular*)

\[
\theta = \frac{\pi}{2} \quad \rightarrow \quad \cos \theta = 0
\]
CASE IV \( \mathbf{A} \cdot \mathbf{B} = -D \quad -AB < -D < 0 \)

The interpretation is some of A is projected onto -B
(A and B point in opposite directions, how much depends on the value of -D)

\[ \frac{\pi}{2} < \theta < \pi \quad \rightarrow \quad -1 < \cos \theta < 0 \]

CASE V \( \mathbf{A} \cdot \mathbf{B} = -AB \) The interpretation is that all of A is projected onto -B
(A and B are anti-parallel: \( \parallel \) but in opposite directions)

\[ \theta = \pi \quad \rightarrow \quad \cos \theta = -1 \]