Collisions

In every collision
1) Conservation of momentum is observed
2) Conservation of total Energy is observed

There are 2 types of collisions:
1) Elastic  (KE is conserved)
2) Inelastic  (KE is NOT conserved)

For inelastic collisions, the missing KE goes to heat, deforming, friction ...

Ex.
Consider 2 objects of mass $m_1$ and $m_2$ colliding elastically. Let $m_1$ have an initial velocity and $m_2$ be initially at rest.

\[ m_1 \quad m_2 \]
\[ \nu_{i1} = \nu_o \quad \nu_{2i} = 0 \]

What are the final velocities of $m_1$ and $m_2$ if:

a) $m_1 = m_2$

b) $m_1 \gg m_2$

c) $m_1 \ll m_2$

From Conservation of momentum

\[ p_i = p_f \]

\[ m_1 \nu_{i1} + m_2 \nu_{2i} = m_1 \nu_{1f} + m_2 \nu_{2f} \]

\[ m_1 \nu_o = m_1 \nu_{1f} + m_2 \nu_{2f} \]
From Conservation of ‘Mechanical’ Energy

\[ E_i = E_f \]

\[ \frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 \]

\[ m_1 v_{o}^2 = m_1 v_{i1}^2 + m_2 v_{i2}^2 \]

Combining the momentum and energy expressions and using a lot of algebra, we find that

\[ v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_o \]

\[ v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_o \]

From these expressions, we can determine the motion of the system by just knowing the initial velocity of \( m_1 \) and both masses.

a) \( m_1 = m_2 \) \hspace{5mm} (2 pool balls)

\[ v_{1i} = v_o \hspace{5mm} v_{1f} = 0 \]

\[ v_{2i} = 0 \hspace{5mm} v_{2f} = v_o \]

Before:

\[ \begin{array}{c}
\text{Before:} \\
\quad m_1 \\
\quad \rightarrow \\
\quad m_2
\end{array} \]

After:

\[ \begin{array}{c}
\text{After:} \\
\quad m_1 \\
\quad \rightarrow \\
\quad m_2
\end{array} \]
b) \( m_1 \gg m_2 \)  
\( (\text{bowling ball hitting a stationary ping pong ball}) \)

\[
\begin{align*}
v_{1i} &= v_o \quad & v_{1f} &\approx v_o \\
v_{2i} &= 0 \quad & v_{2f} &\approx 2v_o 
\end{align*}
\]

Before:

\[
\begin{array}{c}
m_1 \\
\rightarrow
\end{array}
\quad
\begin{array}{c}
m_2
\end{array}
\]

After:

\[
\begin{array}{c}
m_1 \\
\rightarrow
\end{array}
\quad
\begin{array}{c}
m_2
\end{array}
\]

\[
\begin{align*}
\text{Note:} \quad m_2 \text{ will actually move if } m_1 \text{ is not } \ll m_2.
\end{align*}
\]

c) \( m_1 \ll m_2 \)  
\( (\text{ping pong ball hitting a stationary bowling ball}) \)

\[
\begin{align*}
v_{1i} &= v_o \quad & v_{1f} &\approx -v_o \quad (\text{reverses direction}) \\
v_{2i} &= 0 \quad & v_{2f} &\approx 0 
\end{align*}
\]

Before:

\[
\begin{array}{c}
m_1 \\
\rightarrow
\end{array}
\quad
\begin{array}{c}
m_2
\end{array}
\]

After:

\[
\begin{array}{c}
m_1 \\
\leftarrow
\end{array}
\quad
\begin{array}{c}
m_2
\end{array}
\]