Collisions

In every collision involving a closed system:
1) Conservation of momentum is observed
2) Conservation of total Energy is observed

There are 2 types of collisions:
1) Elastic (KE is conserved) \( \Delta KE = 0 \)
2) Inelastic (KE is NOT conserved) \( \Delta KE \neq 0 \)

For inelastic collisions, the missing KE goes to heat, deforming, friction …

Elastic Collision

Ex.
Consider 2 objects of mass \( m_1 \) and \( m_2 \) colliding elastically. Let \( m_1 \) have an initial velocity and \( m_2 \) be initially at rest.

\[
\begin{align*}
\text{m}_1 & \quad \text{m}_2 \\
\nu_{i1} = \nu_o & \quad \nu_{2i} = 0
\end{align*}
\]

What are the final velocities of \( m_1 \) and \( m_2 \) if:

a) \( m_1 = m_2 \)

b) \( m_1 >> m_2 \)

c) \( m_1 << m_2 \)

From Conservation of momentum

\[
\begin{align*}
p_i &= p_f \\
\text{m}_1 \nu_{i1} + \text{m}_2 \nu_{2i} &= \text{m}_1 \nu_{1f} + \text{m}_2 \nu_{2f} \\
\text{m}_1 \nu_o &= \text{m}_1 \nu_{1f} + \text{m}_2 \nu_{2f}
\end{align*}
\]
From Conservation of ‘Mechanical’ Energy

\[ E_i = E_f \]

\[
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
\]

\[ m_1 v_o^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \]

Combining the momentum and energy expressions and using a lot of algebra, we find that

\[ v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_o \]

\[ v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_o \]

From these expressions, we can determine the motion of the system based on the initial velocity of \( m_1 \) and the ratio of the masses.

\( a) \quad m_1 = m_2 \quad (2 \text{ pool balls}) \)

\[ v_{1i} = v_o \quad v_{1f} = 0 \]

\[ v_{2i} = 0 \quad v_{2f} = v_o \]

Before:

\[ \begin{array}{c}
\text{\[ m_1 \quad \rightarrow \quad m_2 \]}
\end{array} \]

After:

\[ \begin{array}{c}
\text{\[ m_1 \quad \rightarrow \quad m_2 \]}
\end{array} \]
b) \[ m_1 >> m_2 \] 
*(bowling ball hitting a stationary ping pong ball)*

\[
\begin{align*}
  v_{1i} &= v_o \\
  v_{2i} &= 0
\end{align*}
\]

\[
\begin{align*}
  v_{1f} &\approx v_o \\
  v_{2f} &\approx 2v_o
\end{align*}
\]

**Before:**

\[m_1 \quad \quad m_2\]

**After:**

\[m_1 \quad \quad m_2\]

---

c) \[ m_1 << m_2 \] 
*(ping pong ball hitting a stationary bowling ball)*

\[
\begin{align*}
  v_{1i} &= v_o \\
  v_{2i} &= 0
\end{align*}
\]

\[
\begin{align*}
  v_{1f} &\approx -v_o \quad (reverses \ direction) \\
  v_{2f} &\approx 0
\end{align*}
\]

**Before:**

\[m_1 \quad \quad m_2\]

**After:**

\[m_1 \quad \quad m_2\]

---

**Note:** \( m_2 \) will actually move if \( m_1 \) is not \( << m_2 \).
Ex.

Consider 2 objects of mass \( m_1 \) and \( m_2 \) colliding elastically. Let \( m_1 \) have an initial velocity toward \( m_2 \) and \( m_2 \) have an initial velocity toward \( m_1 \).

What are the final velocities of \( m_1 \) and \( m_2 \) in terms of \( v_{1i} \) and \( v_{2i} \)?

**From Conservation of momentum**

\[ p_i = p_f \]

\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{(minus signs for v will be added later)} \]

**From Conservation of ‘Mechanical’ Energy**

\[ E_i = E_f \]

\[ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \]

**After combining the 2 expressions and much algebra, we get:**

\[ v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{2i} \]

\[ v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i} \]

**Note:** If \( v_{2i} = 0 \), we get back our previous examples expressions for the final velocities:

\[ v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} \]

\[ v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} \]
What are the final velocities of \( m_1 \) and \( m_2 \) if \( \nu_o > \nu'_o \) &:

\( a \) \( m_1 = m_2 \)

\( b \) \( m_1 \gg m_2 \)

\( c \) \( m_1 \ll m_2 \)

\( a \) \( m_1 = m_2 \) (2 pool balls)

\[ \nu_{1i} = \nu_o \quad \nu_{1f} = -\nu'_o \]

\[ \nu_{2i} = -\nu'_o \quad \nu_{2f} = \nu_o \quad (reverses \ direction) \]

Before:

\[ \begin{array}{c}
\text{Before:} \\
\quad m_1 \\
\quad m_2
\end{array} \]

After:

\[ \begin{array}{c}
\text{After:} \\
\quad m_1 \\
\quad m_2
\end{array} \]

\( b \) \( m_1 \gg m_2 \) (bowling ball hitting a slow moving ping pong ball)

\[ \nu_{1i} = \nu_o \quad \nu_{1f} \approx \nu_o \]

\[ \nu_{2i} = -\nu'_o \quad \nu_{2f} \approx 2\nu_o + \nu'_o \quad (reverses \ direction) \]

Before:

\[ \begin{array}{c}
\text{Before:} \\
\quad m_1 \\
\quad m_2
\end{array} \]

After:

\[ \begin{array}{c}
\text{After:} \\
\quad m_1 \\
\quad m_2
\end{array} \]
c) \( m_1 \ll m_2 \)  

*ping pong ball hitting a slow moving bowling ball*

\[
\begin{align*}
v_{1i} &= v_o \\
v_{1f} &\approx -v_o - 2v'_o \quad \text{(reverses direction)}
\end{align*}
\]

\[
\begin{align*}
v_{2i} &= -v'_o \\
v_{2f} &\approx -v'_o
\end{align*}
\]

**Before:**

![Diagram showing the initial state of the collision]

**After:**

![Diagram showing the final state of the collision]

**Inelastic Collision**

Ex.

Consider 2 objects of mass \( m_1 \) and \( m_2 \) colliding inelastic-ly. Let \( m_1 \) have an initial velocity and \( m_2 \) be initially at rest and afterward, they both move stuck together with the same velocity.

\[
\begin{align*}
v_{1i} &= v_o \\
v_{2i} &= 0 \\
v_f &=
\end{align*}
\]

**Before**

![Diagram showing the initial state of the collision]

**After**

![Diagram showing the final state of the collision]

**From Conservation of momentum**

\[
\begin{align*}
p_i &= p_f \\
m_1v_{1i} &= (m_1 + m_2)v_f
\end{align*}
\]
From Conservation of ‘Mechanical’ Energy

\[ E_i = E_f \]

\[ \frac{1}{2} m_1 v_{i1}^2 = \frac{1}{2} (m_1 + m_2) v_{1f}^2 \]

Combining these 2 expressions and after a little algebra:

\[ \frac{(m_1 + m_2)}{m_1} v_f^2 = (m_1 + m_2) v_{1f}^2 \]

\[ m_1 + m_2 = m_1 \quad \text{This statement is False!!!} \]

Ex.

Let \( m_1 = m \) \& \( m_2 = 2m \)

From the previous expression, this would yield

\[ m + 2m = m \]

\[ 3m = m \quad ?!?!!? \]

\[ 3 = 1 \]

This means that KE was NOT conserved in the inelastic collision. The problem could be fixed using conservation of ‘total’ energy:

\[ \frac{1}{2} m_1 v_{i1}^2 = \frac{1}{2} (m_1 + m_2) v_{1f}^2 + E_{loss} \]

Now we would have to come up with some way to represent \( E_{loss} \).