Cars Traveling Around a Banked Curve \( (\text{no friction}) \)

**NOTE:** In these types of problems, always choose a coordinate system such that \( x \) is parallel or antiparallel to \( a_r \).

**Ex.**

Find the exact speed a car of mass \( m \) traveling along a banked curve (whose path is the shape of a circle of radius \( r \)) must have in order to make the curve without sliding up or down the incline.

Determine the motion in each direction using Newton’s 2\(^{\text{nd}}\) law and the force diagram.

\[
\sum F_x = -ma_r \\
-N_x = -ma_r \\
\sum F_y = 0 \\
N_y - W = 0
\]

Substituting & solving for the Normal Force:

\[
-N \sin \theta = -m \left( \frac{v^2}{r} \right) \\
N \cos \theta - mg = 0
\]

\[N = \frac{mv^2}{r \sin \theta} \quad \text{and} \quad N = \frac{mg}{\cos \theta}\]

Equating the two expressions for the Normal force:

\[\frac{mv^2}{r \sin \theta} = \frac{mg}{\cos \theta}\]

\[v^2 = \frac{rg \sin \theta}{\cos \theta}\]

\[v = \sqrt{rg \tan \theta}\]

\( W/o \, \text{friction, this is the restriction on the speed of the car to go around a banked curve without sliding up or down the incline.} \)

If \( v_{\text{car}} > v \), the car will slide up the incline.

If \( v_{\text{car}} < v \), the car will slide down the incline.

In terms of the angle:

\[\tan \theta = \frac{v^2}{rg}\]