Work done on a box by external forces

* What is the work done on the box by external forces if it moves at a constant speed \( v_0 \) and:

\[
\begin{align*}
d & = 250 \text{ m} \\
m & = 210 \text{ kg} \\
\theta & = 10^\circ
\end{align*}
\]

For this problem, we will find the net force acting on the box in the direction of motion and then use \( W = \vec{F}_{net} \cdot \vec{d} \).

To find \( \vec{F}_{net} \) in the direction of motion, use Newton’s Laws:

\[
\begin{align*}
\sum F_x &= 0 \quad (\text{moves @ constant speed } \rightarrow a = 0) & \sum F_y &= 0 \quad (\text{no vertical motion}) \\
F \cos \theta - f_k &= 0 & N - w + F \sin \theta &= 0 \\
& N = mg - F \sin \theta
\end{align*}
\]

Using \( f_k = \mu_k N \)

\[
F \cos \theta - \mu_k (mg - F \sin \theta) = 0
\]

\[
F = \frac{\mu_k mg}{\cos \theta + \mu_k \sin \theta}
\]
Now that we know the net force, the work done on the box can be calculated using:

\[ W = \vec{F}_{net} \cdot \vec{d} \]

\[ W = Fd \cos \theta \]

\[ W = \left( \frac{\mu_k mg}{\cos \theta + \mu_k \sin \theta} \right) d \cos \theta \]

Inserting our numbers:

\[ W = \left( \frac{(0.2)(210 \text{ kg})(9.8 \text{ m/s}^2)}{\cos(10^\circ) + (0.2)\sin(10^\circ)} \right) (250 \text{ m})\cos(10^\circ) \]

\[ W = 99,000 \text{ J} \]