**Ex. Electric Field due to a Finite Charged Rod**

Find the electric field some distance \( y \) above a uniformly charged finite rod

![Diagram of a finite charged rod with electric field lines](image)

The total length of the rod is \( L (L = 2l) \).

Since this is a uniform charged rod \( \rightarrow \lambda = \frac{dq}{dl} \rightarrow dq = \lambda dx_i \)

At point \( P \):

\[
dE_i = \frac{k_e |dq_i|}{r^2} \hat{r} = \frac{k_e |dq_i|}{r^2} (\sin \theta \hat{x} + \cos \theta \hat{y}) = \left( \frac{k_e dq_i}{r^2} \sin \theta \right) \hat{x} + \left( \frac{k_e dq_i}{r^2} \cos \theta \right) \hat{y}
\]

The total electric field \( E \) due to \( dq_i \) can be found by summing up (integrating) all the \( dq_i \) contributions for the entire length of the rod:

\[
E = \int_{-l}^{l} \left( \frac{k_e dq}{r^2} \sin \theta \right) \hat{x} + \left( \frac{k_e dq}{r^2} \cos \theta \right) \hat{y}
\]

Substituting for \( dq, r, \sin \theta \) and \( \cos \theta \) and separating the integral into its two components we get:

\[
E = \int_{-l}^{l} \left( \frac{k_e x \lambda dx}{(x^2 + y^2)^{3/2}} \right) \hat{x} + \int_{-l}^{l} \left( \frac{k_e y \lambda dx}{(x^2 + y^2)^{3/2}} \right) \hat{y}
\]
The first integral (\(\hat{x}\) direction) is an odd function \[ f(-x) = -f(x) \]. Odd functions evaluated over symmetric limits always integrate to 0.

\[ \mathbf{E} = 0 + \int_{-l}^{l} \left( \frac{k_y \lambda dx}{(x^2 + y^2)^{3/2}} \right) \hat{y} \]

Rearranging the limits of the integral:

\[ \mathbf{E} = 2 \int_{0}^{l} \left( \frac{k_y \lambda dx}{(x^2 + y^2)^{3/2}} \right) \hat{y} \]

Integrating yields:

\[ \mathbf{E} = 2k_y \lambda y \left[ \frac{x}{y^2 \sqrt{y^2 + x^2}} \right]_{0}^{l} \hat{y} \]

\[ \mathbf{E} = \frac{2k_y \lambda l}{y \sqrt{y^2 + l^2}} \hat{y} \]

This implies that the electric field is always perpendicular to the center of the wire, since there is no \(x\) component in the final expression for \(\mathbf{E}\).